Visual Exploration of the Effect of Constraint Handling in Multiobjective Optimization

Supplementary Material

Tea Tušar[1,2][0000−0002−6495−006X], Aljoša Vodopija[1,2][0000−0003−0299−4160], and Bogdan Filipič[1,2][0000−0003−4428−4255]

[1] Jožef Stefan Institute, Ljubljana, Slovenia
[2] Jožef Stefan International Postgraduate School, Ljubljana, Slovenia
{tea.tusar,aljosa.vodopija,bogdan.filipic}@ijs.si

Abstract. This is the supplementary material for the paper *Visual Exploration of the Effect of Constraint Handling in Multiobjective Optimization* accepted for publication at the EMO 2023 conference. It provides the definitions of the constraint function for problems CBB1–4 and the landscape visualizations for the 12 constrained multiobjective optimization problems considered in the study.

Keywords: Constrained multiobjective optimization · Constraint handling technique · Problem landscape · Visualization

1 Definitions of Test Problems CBB1–4

All four CBB problems were created by adding different constraints to the first instance of the 2-D `bbob-biobj` problem F_1 (the double sphere problem) [1]. In the following, we provide the definitions of the constraint function for each of these problems:

$$
\begin{aligned}
g_{\mathrm{CBB1}}(x) &= 0.2x_1 + x_2 + 2\\
g_{\mathrm{CBB2}}(x) &= -0.01 + f_{\mathrm{PDF}}\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}, \begin{bmatrix}-2\\-1\end{bmatrix}, \begin{bmatrix}0 & 0.5\\0.5 & 0\end{bmatrix}\right)\\
g_{\mathrm{CBB3}}(x) &= -0.01 + f_{\mathrm{PDF}}\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}, \begin{bmatrix}-2\\-1\end{bmatrix}, \begin{bmatrix}0 & 5\\5 & 0\end{bmatrix}\right)\\
g_{\mathrm{CBB4}}(x) &= 0.01 - \max\Bigg\{ f_{\mathrm{PDF}}\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}, \begin{bmatrix}2\\-3\end{bmatrix}, \begin{bmatrix}0 & 5\\5 & 0\end{bmatrix}\right),\\
&\qquad\qquad f_{\mathrm{PDF}}\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}, \begin{bmatrix}-3\\2\end{bmatrix}, \begin{bmatrix}0 & 0.4\\0.4 & 0\end{bmatrix}\right),\\
&\qquad\qquad f_{\mathrm{PDF}}\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}, \begin{bmatrix}3\\3\end{bmatrix}, \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\right)\Bigg\},
\end{aligned}
$$

where f_{PDF} is the probability density function of the multivariate normal distribution defined as:

$$f_{\text{PDF}}(x, \mu, \Sigma) = \frac{1}{\sqrt{(2\pi)^n \det \Sigma}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right),$$

where μ is the mean, Σ the covariance matrix and n the dimension of x.

2 Landscape Visualizations for 12 Constrained Multiobjective Optimization Problems

We present the problem landscape and local search result plots for all 12 constrained multiobjective optimization problems (CMOPs) and all six constraint handling techniques (CHTs) considered in the corresponding paper. The figures follow the order of the problems as presented in Table 1.

Table 1. CMOPs categorized by their type.

	Type I problems	
DAS-CMOP3 Fig. 1	DAS-CMOP5 Fig. 2	MW14 Fig. 3
	Type II problems	
C2-DTLZ2 Fig. 4	DAS-CMOP1 Fig. 5	DC1-DTLZ1 Fig. 6
	Type III problems	
CBB1 Fig. 7	CBB2 Fig. 8	MW3 Fig. 9
	Type IV problems	
CBB3 Fig. 10	CBB4 Fig. 11	MW11 Fig. 12

References

1. Brockhoff, D., Auger, A., Hansen, N., Tušar, T.: Using well-understood single-objective functions in multiobjective black-box optimization test suites. Evolutionary Computation **30**(2), 165–193 (2022)

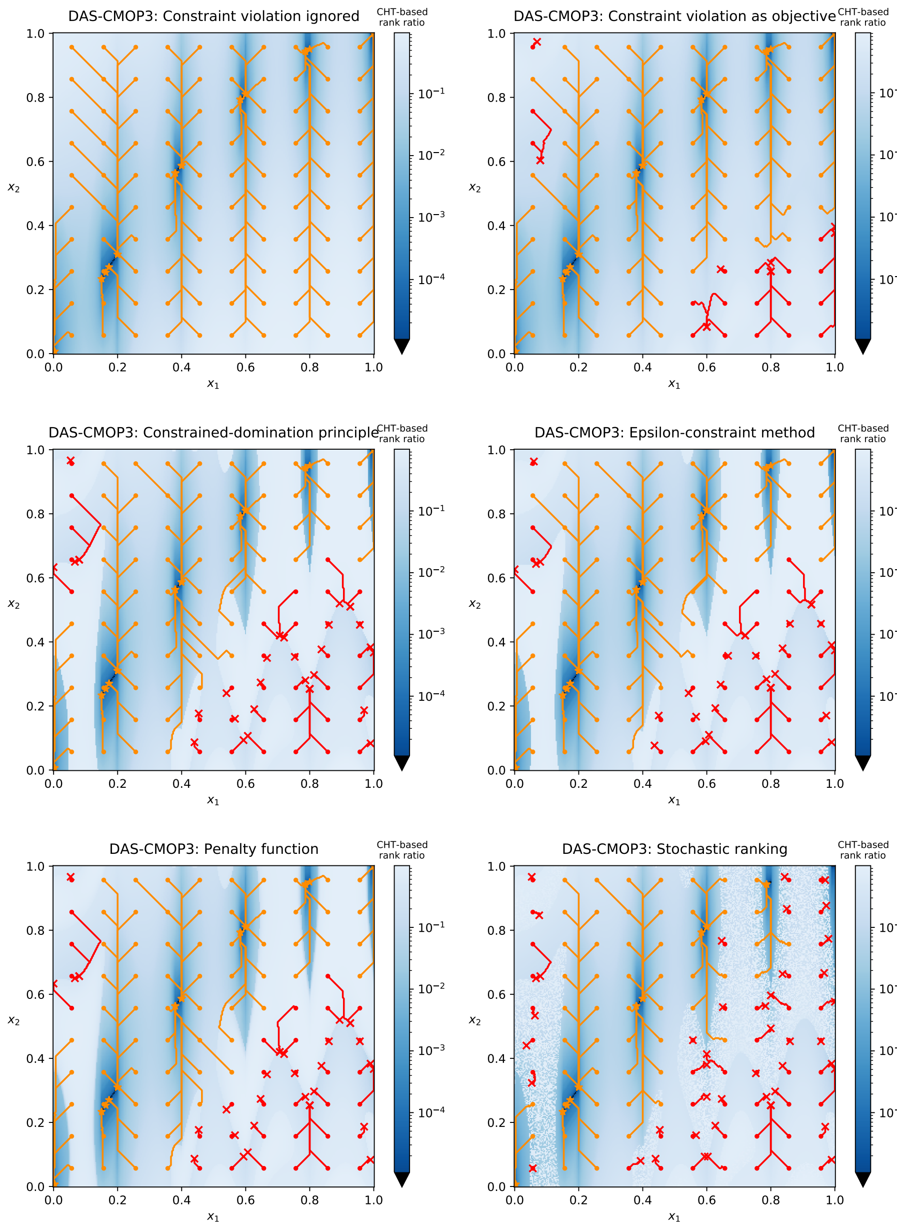

Fig. 1. Plots of the CHT-based landscapes for Type I problem DAS-CMOP3 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

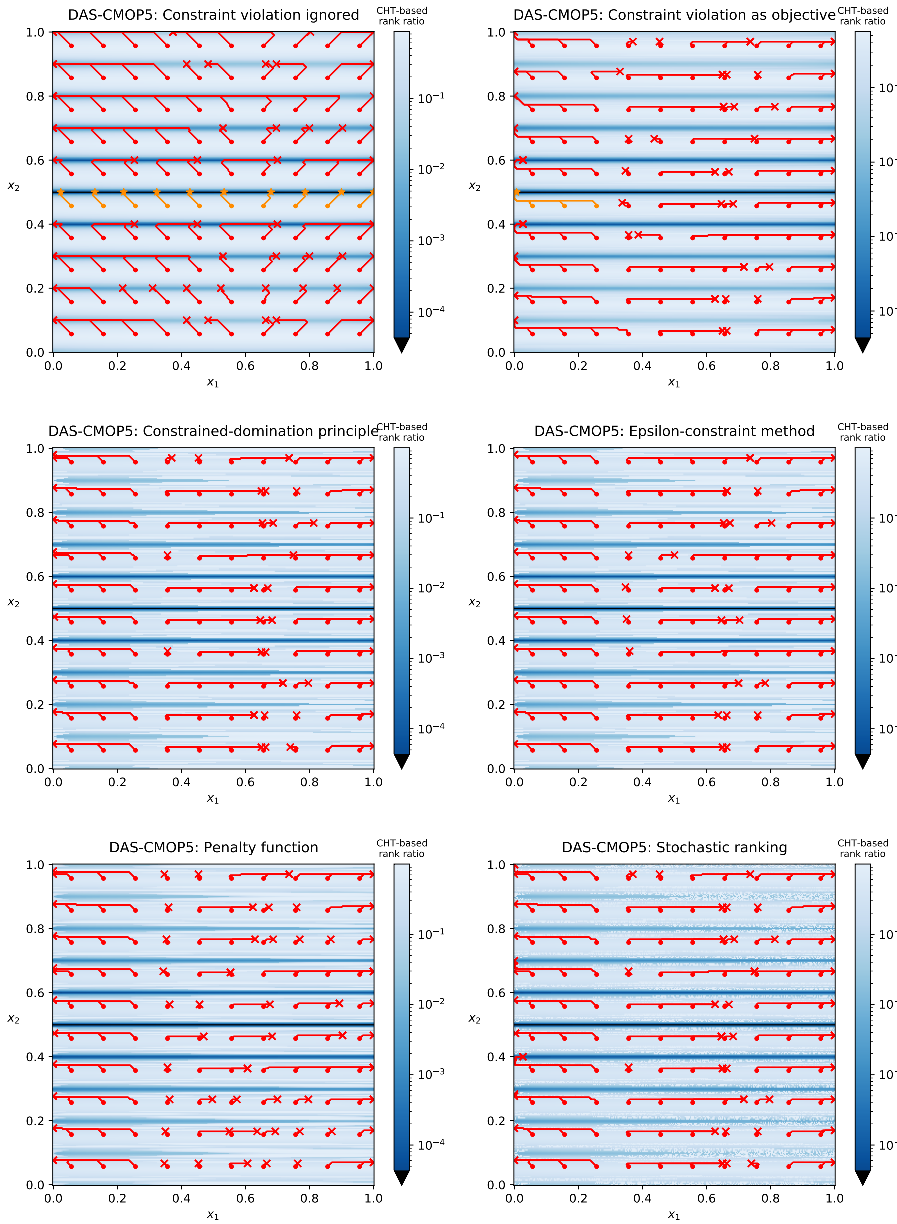

Fig. 2. Plots of the CHT-based landscapes for Type I problem DAS-CMOP5 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

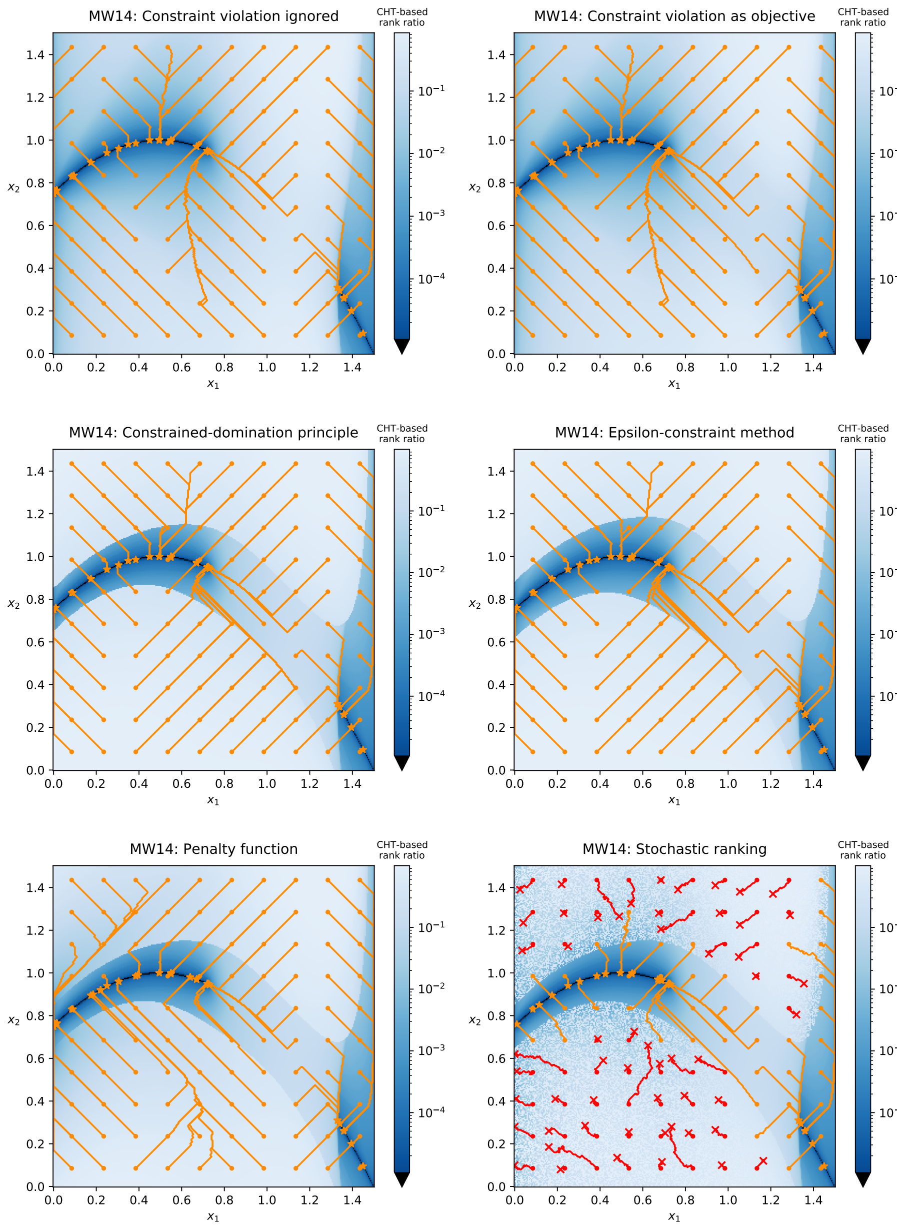

Fig. 3. Plots of the CHT-based landscapes for Type I problem MW14 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

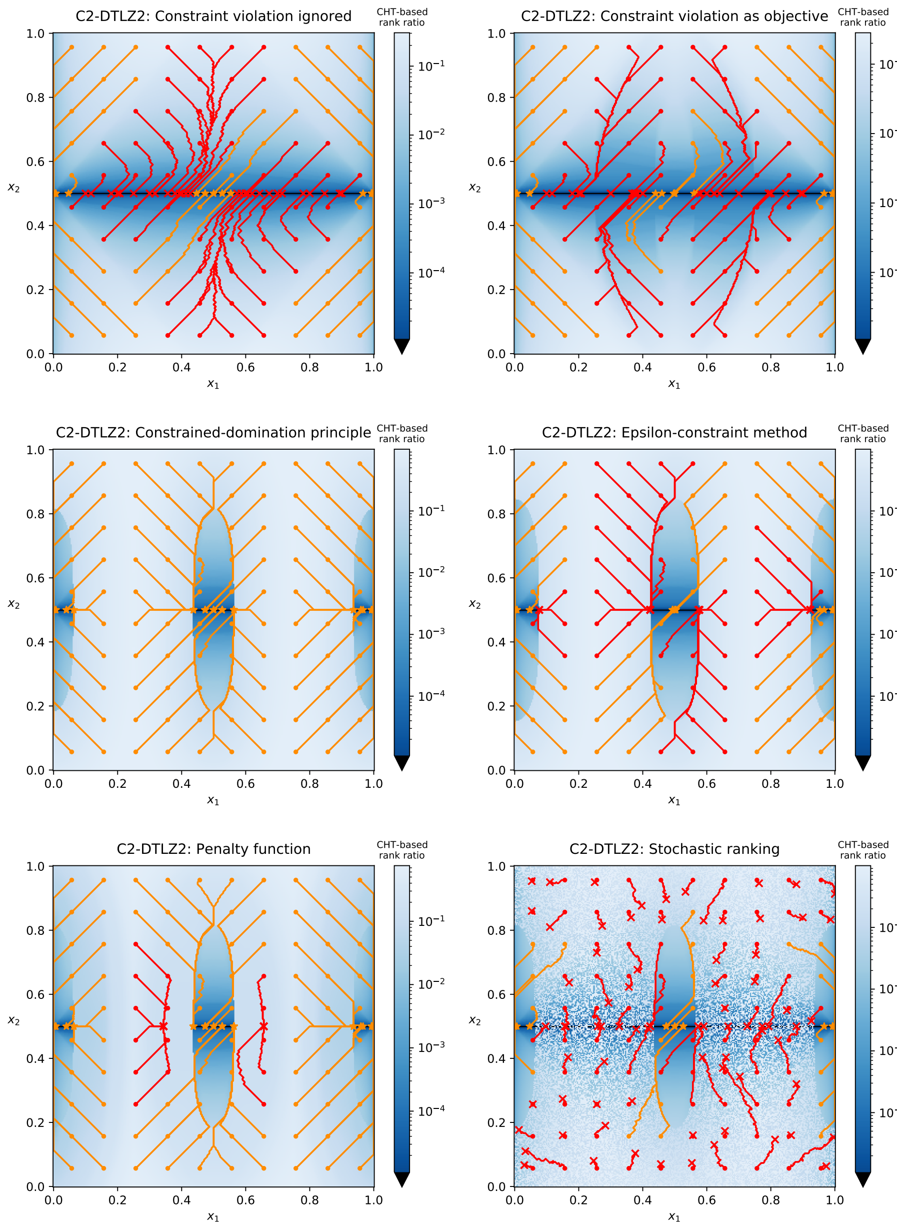

Fig. 4. Plots of the CHT-based landscapes for Type II problem C2-DTLZ2 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

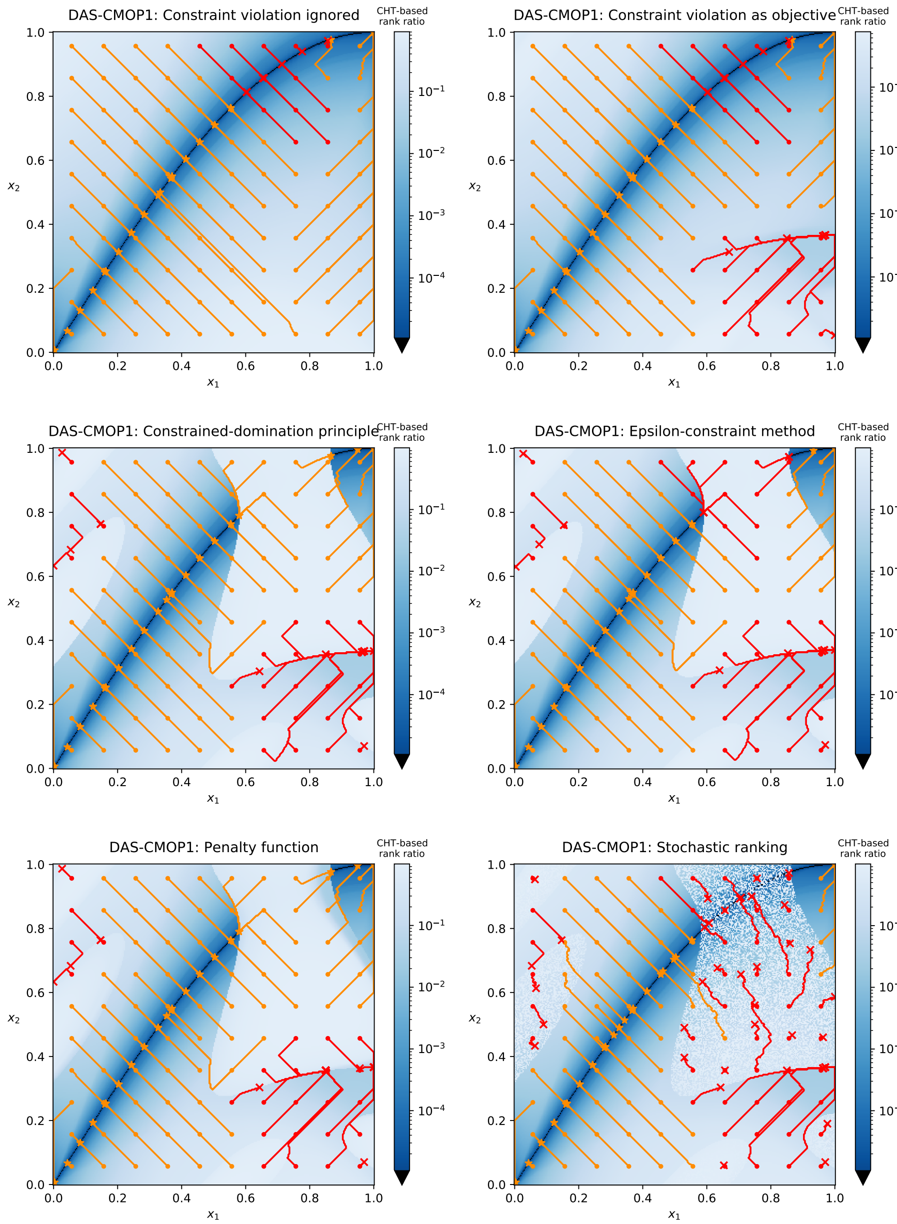

Fig. 5. Plots of the CHT-based landscapes for Type II problem DAS-CMOP1 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

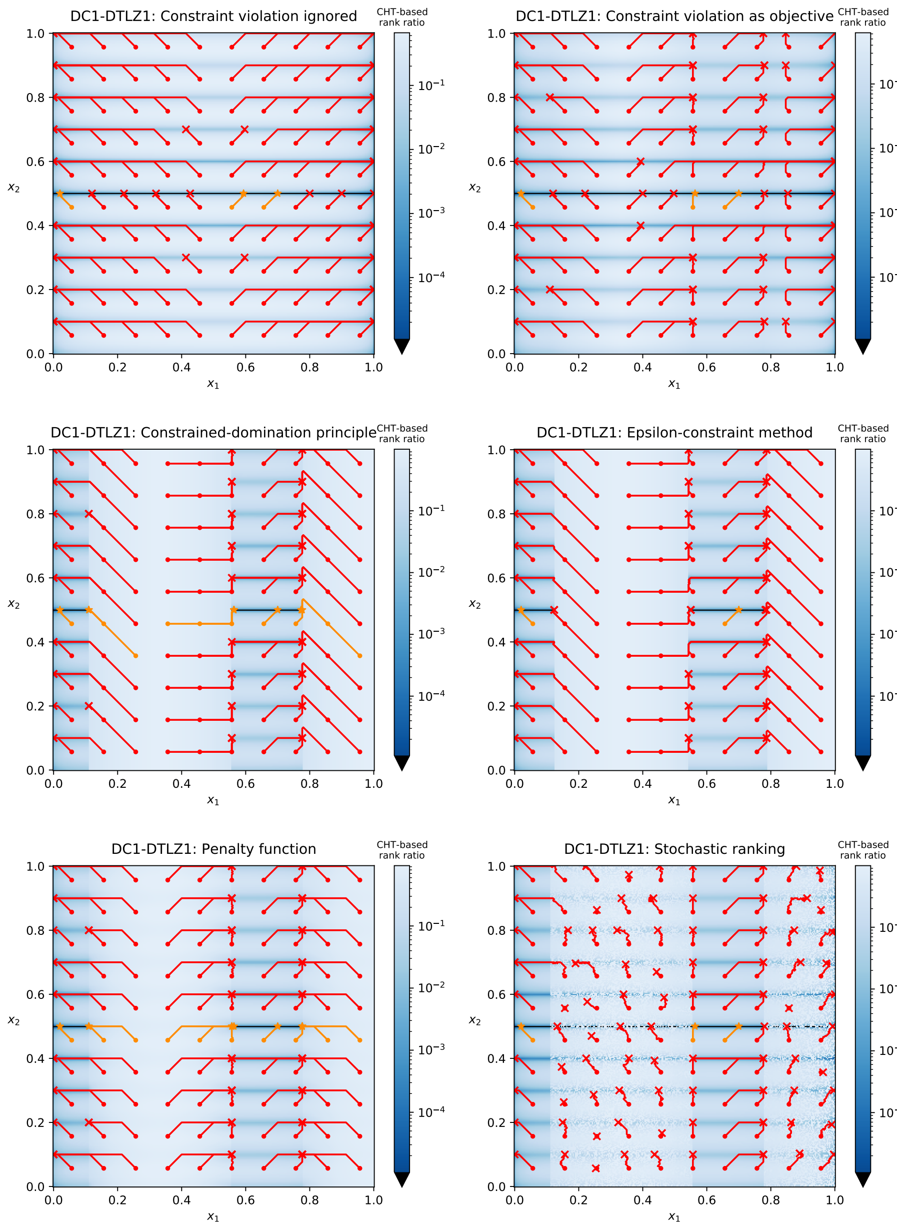

Fig. 6. Plots of the CHT-based landscapes for Type II problem DC1-DTLZ1 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

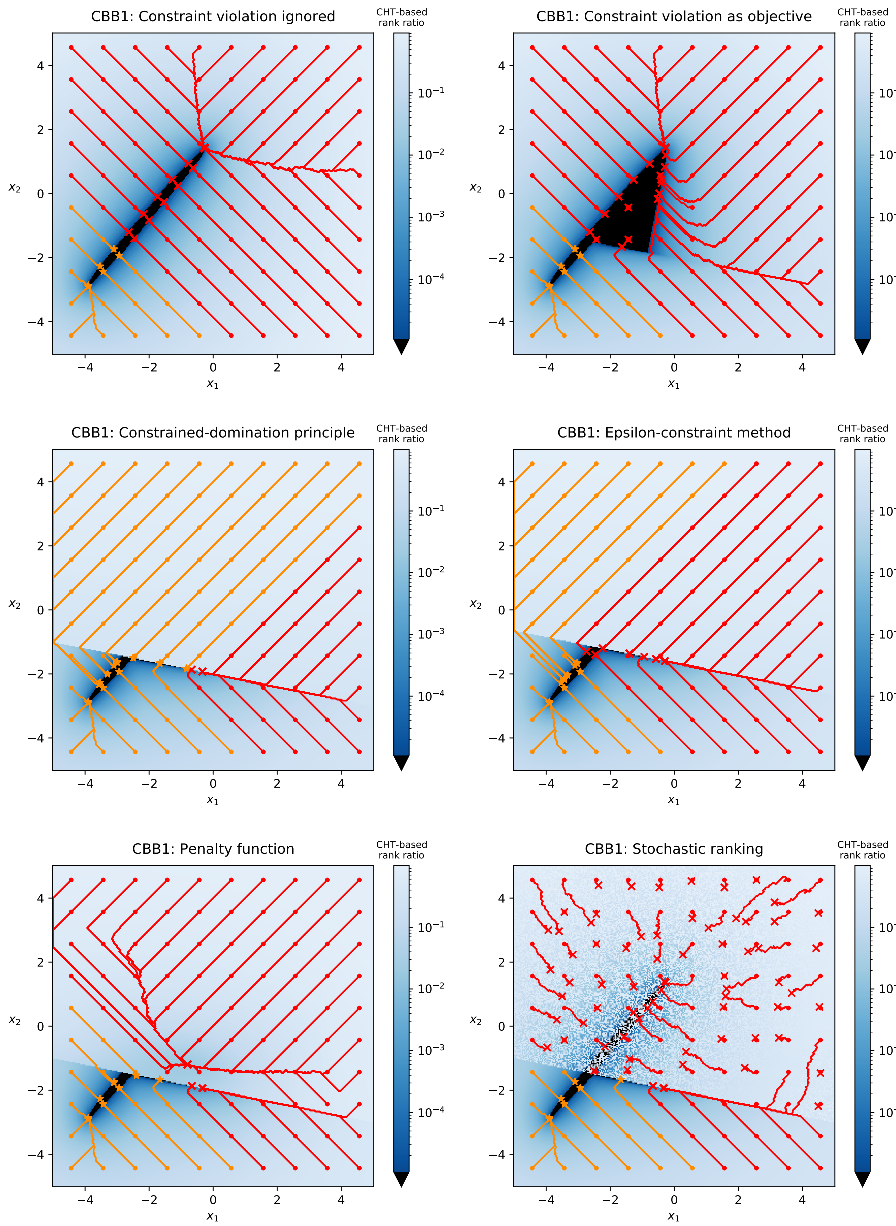

Fig. 7. Plots of the CHT-based landscapes for Type III problem CBB1 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

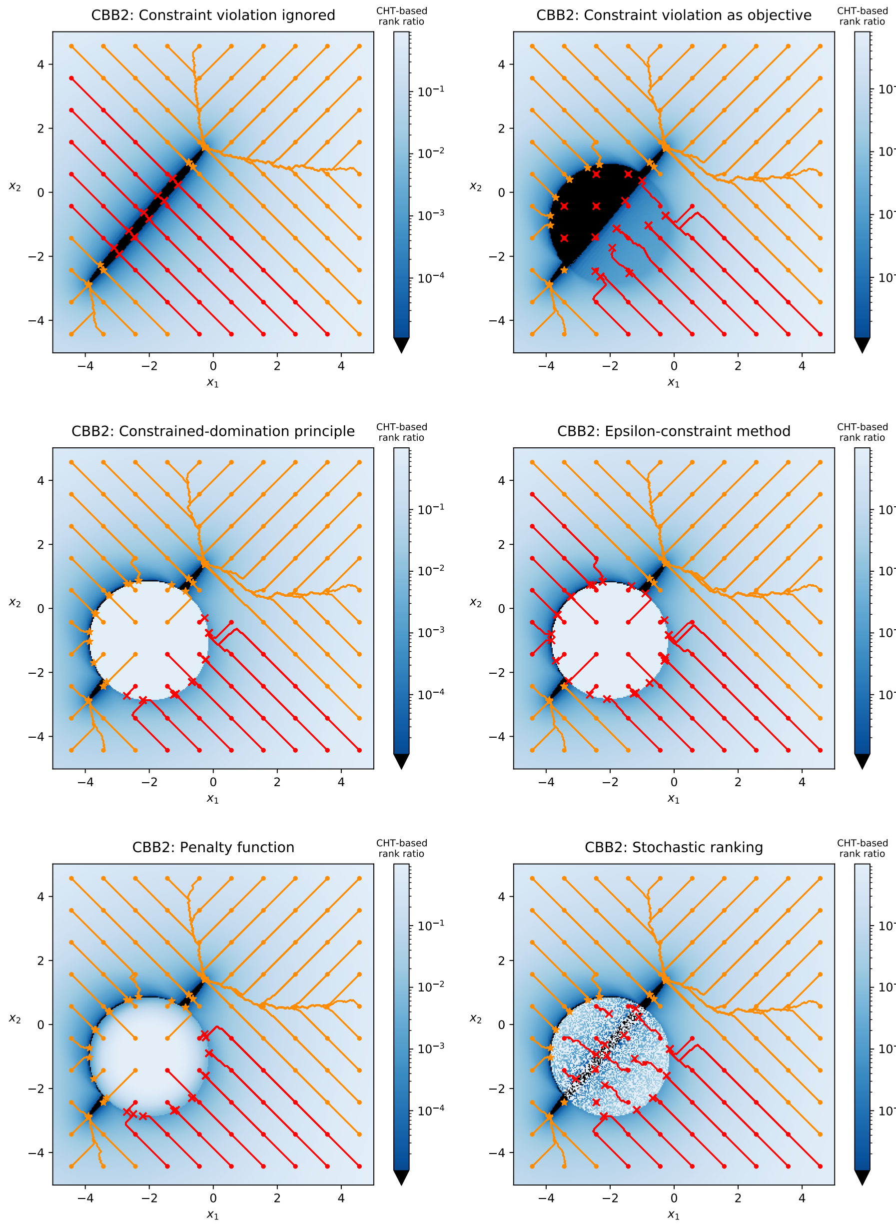

Fig. 8. Plots of the CHT-based landscapes for Type III problem CBB2 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

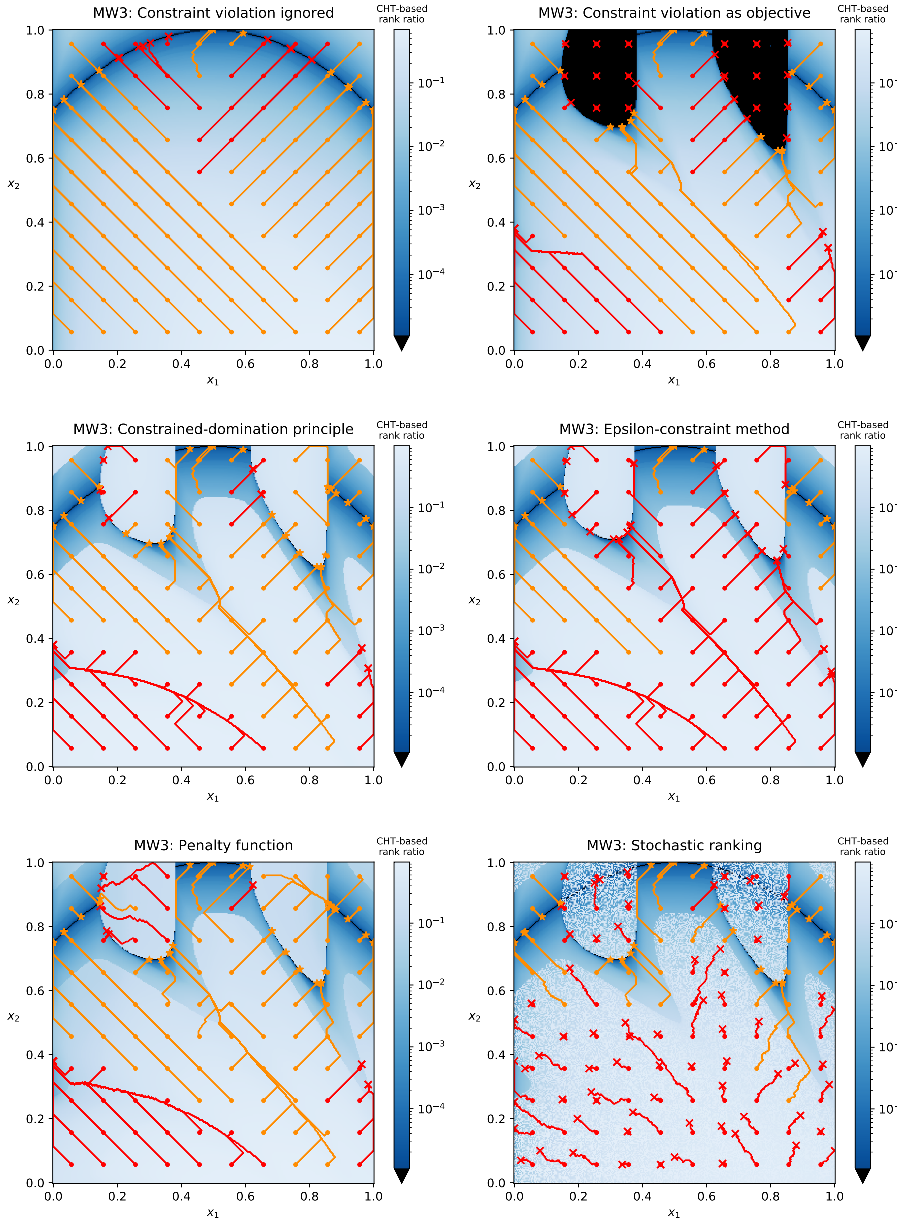

Fig. 9. Plots of the CHT-based landscapes for Type III problem MW3 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

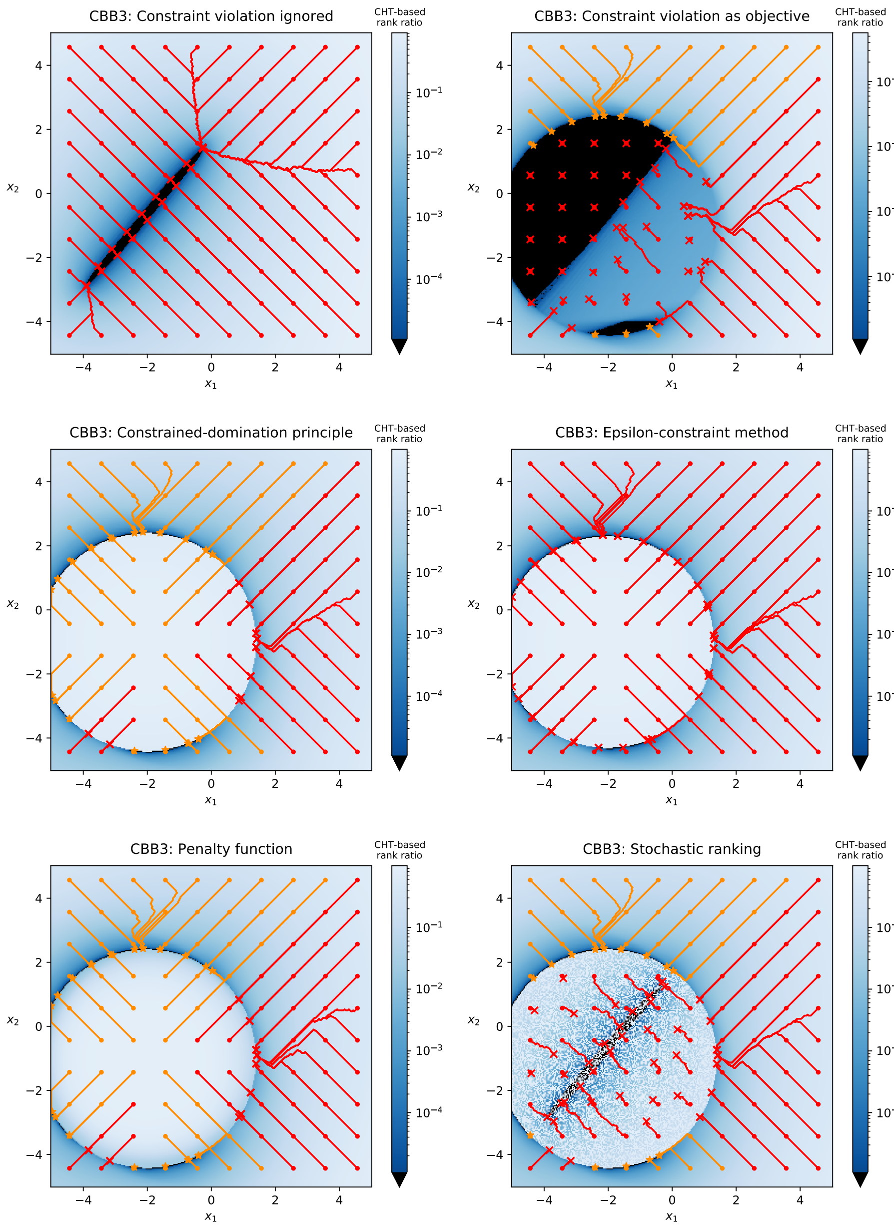

Fig. 10. Plots of the CHT-based landscapes for Type IV problem CBB3 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

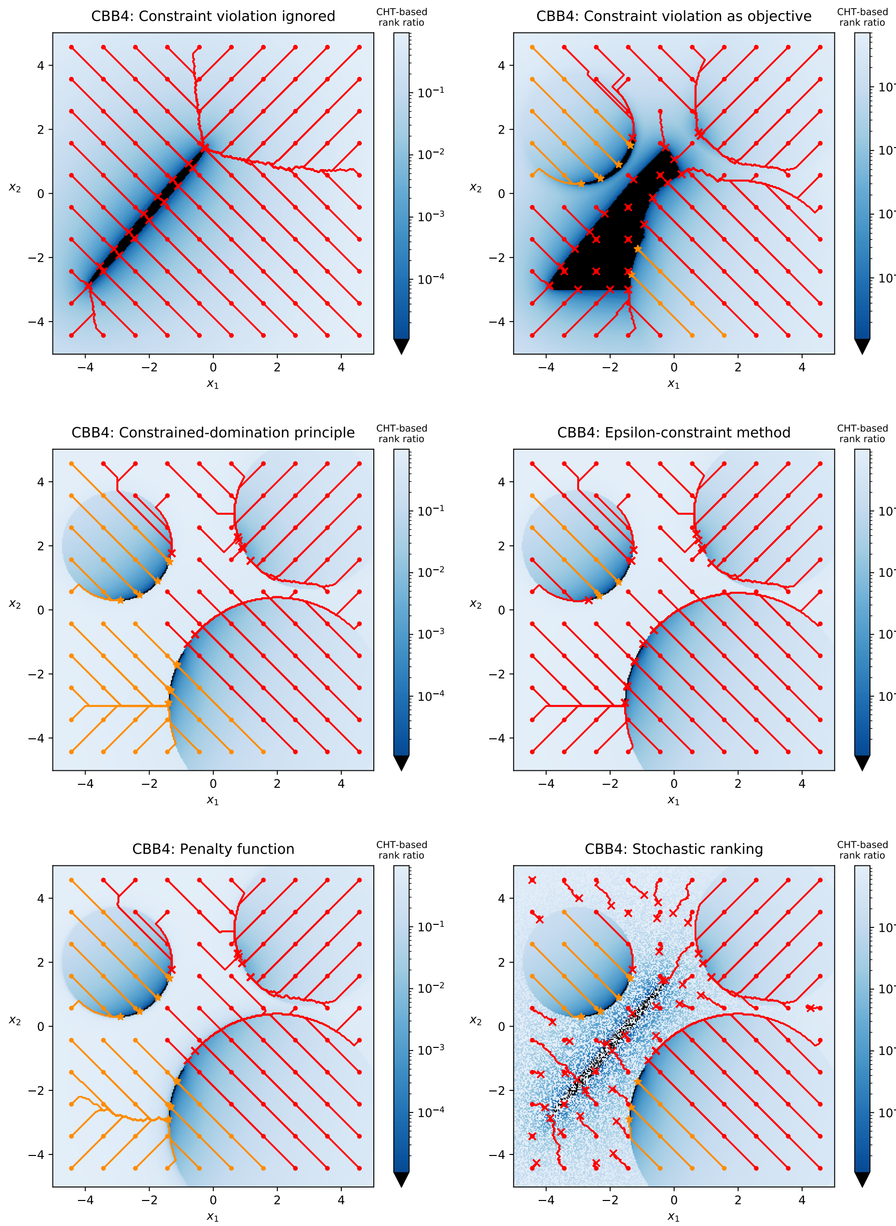

Fig. 11. Plots of the CHT-based landscapes for Type IV problem CBB4 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.

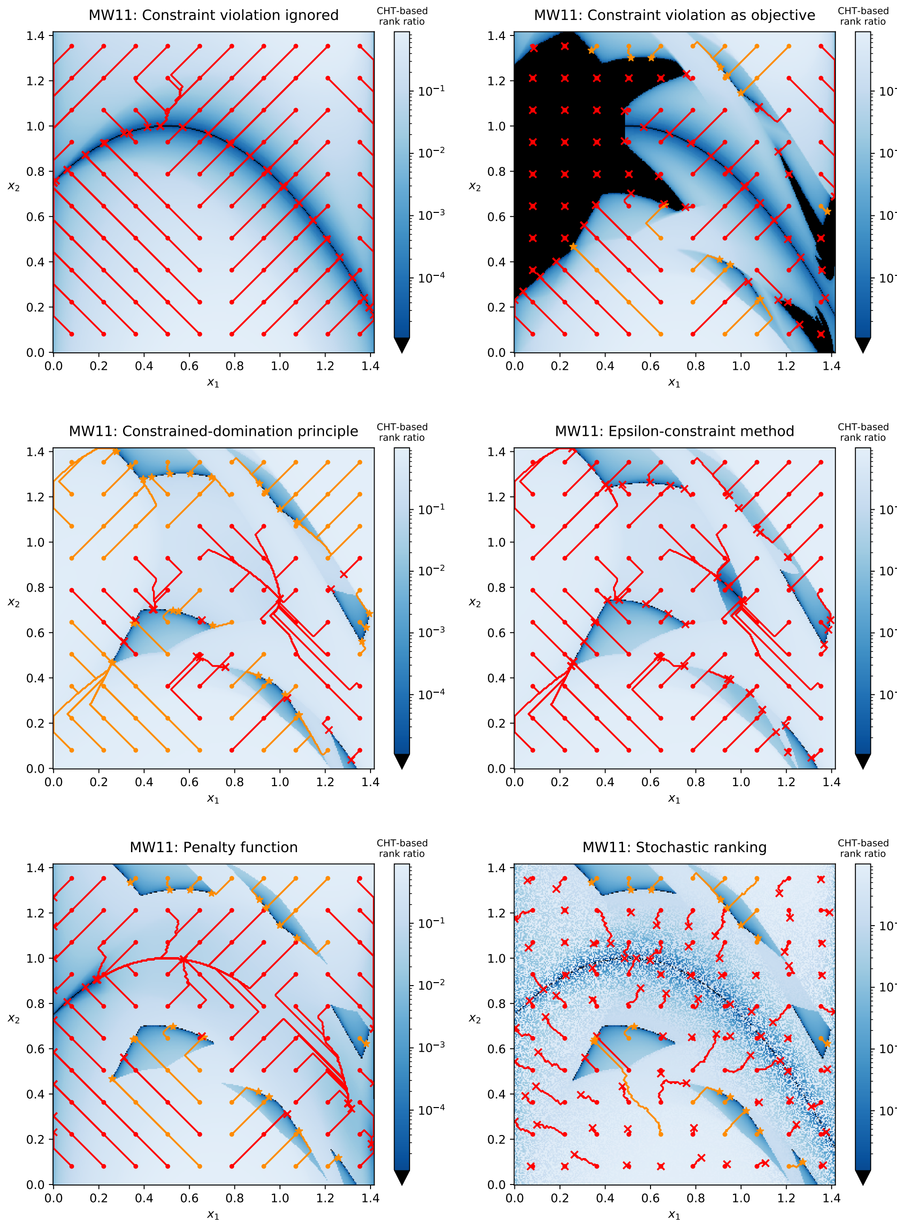

Fig. 12. Plots of the CHT-based landscapes for Type IV problem MW11 (in blue hues) for the six considered CHTs. Black denotes the Pareto set of these landscapes. Orange and red lines show the paths of local optimization starting in 100 different points shown with dots. If the path ends in a point that is optimal in the original problem landscape, the line is orange and it ends with a star, otherwise the line is red and it ends with a cross.