Showing the Knee of a 4-D Pareto Front Approximation via Different Visualization Methods

Tea Tušar
Department of Intelligent Systems
Jožef Stefan Institute
Ljubljana, Slovenia
tea.tusar@ijs.si

Bogdan Filipič
Department of Intelligent Systems
Jožef Stefan Institute
Ljubljana, Slovenia
bogdan.filipic@ijs.si

ABSTRACT

When decision makers select one or more trade-off solutions to a multiobjective optimization problem, they are mostly interested in solutions residing at knees—regions of the Pareto front where a small improvement in one objective leads to a large deterioration in at least one other objective. It is therefore important to be able to detect such regions, preferably through visualization. This paper presents visualizations of Pareto front approximations of a multiobjective problem with knees. More specifically, we show how a sampled Pareto front of a four-objective problem with a single knee looks like when visualized using seven different methods (scatter plot matrix, bubble chart, parallel coordinates, radial coordinate visualization, level diagrams, hyper-radial visualization and prosections). We can observe that while the first four methods cannot visualize the knee, the remaining three are able to do so and are therefore more suitable for use in visualization of Pareto front approximations.

1. INTRODUCTION

When inspecting a set of solutions to a multiobjective optimization problem, decision makers usually prefer solutions lying at knees—regions of the Pareto front where a small improvement in one objective leads to a large deterioration in at least one other objective [2]. Visualizing knee regions in a clear and concise way is therefore a crucial requirement for supporting the decision making process. Finding a good representation of the Pareto front and its knees is rather straightforward in the case of two or three objectives (see Figures 1 and 2 for two examples)[1], but very challenging when the number of objectives is four or more [8].

This paper presents visualizations of the single-knee instance of the DEB4DK problem [2, 7] using seven different methods, namely the scatter plot matrix, bubble chart, parallel coordinates [5], radial coordinate visualization [4], level diagrams [1], hyper-radial visualization [3] and prosections [8]. All these methods have been previously used to visualize Pareto front approximations and have been analyzed with respect to some desired properties for visualization methods [8].

Section 2 provides the formal definition of the DEB4DK problem, while the visualizations with different methods are shown in Section 3. Some concluding remarks are presented in Section 4.

[1]Minimization in all objectives is assumed throughout this paper.

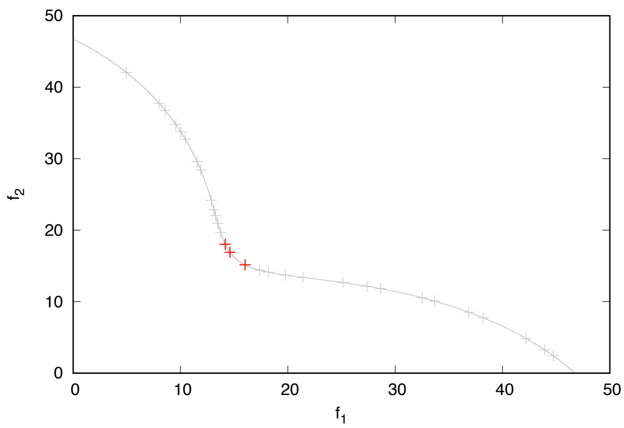

Figure 1: The DEB2DK problem with a single knee (solutions near the knee are shown in red).

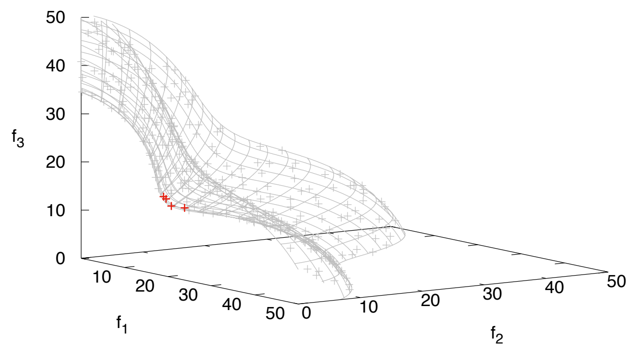

Figure 2: The DEB3DK problem with a single knee (solutions near the knee are shown in red).

2. THE DEB4DK PROBLEM

Branke et al. introduced a family of optimization problems with knees that are based on the DTLZ benchmark problems and scalable to any number of objectives [2]. While only the instances with two and three objectives were presented originally, they were scaled to four and five objectives in [7].

Let us recall the formal definition of the DEB4DK problem with four objectives:

$$
\begin{aligned}
f_1(\boldsymbol{x}) &= g(\boldsymbol{x})r(\boldsymbol{x}) \sin\left(\frac{\pi}{2}x_1\right) \sin\left(\frac{\pi}{2}x_2\right) \sin\left(\frac{\pi}{2}x_3\right) \\
f_2(\boldsymbol{x}) &= g(\boldsymbol{x})r(\boldsymbol{x}) \sin\left(\frac{\pi}{2}x_1\right) \sin\left(\frac{\pi}{2}x_2\right) \cos\left(\frac{\pi}{2}x_3\right) \\
f_3(\boldsymbol{x}) &= g(\boldsymbol{x})r(\boldsymbol{x}) \sin\left(\frac{\pi}{2}x_1\right) \cos\left(\frac{\pi}{2}x_2\right) \\
f_4(\boldsymbol{x}) &= g(\boldsymbol{x})r(\boldsymbol{x}) \cos\left(\frac{\pi}{2}x_1\right) \\
g(\boldsymbol{x}) &= 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i \\
r(\boldsymbol{x}) &= \frac{r_1(x_1) + r_2(x_2) + r_3(x_3)}{3} \\
r_i(x_i) &= 5 + 10(x_i - 0.5)^2 + \frac{3\cos(2K\pi x_i)}{K} \\
\boldsymbol{x} &\in [0,1]^n.
\end{aligned}
$$

Here, n denotes the dimensionality of the decision space, while parameter K is used to control the number of knees on the Pareto front. A DEBmDK optimization problem with m objectives has K^{m-1} knees. This means that the four-objective DEB4DK problem can be instantiated to have $1, 8, 27, \ldots$ knees (for $K = 1, 2, 3, \ldots$ respectively). In the rest of the paper, the value $K = 1$ yielding the Pareto front with a single knee will be used.

3. RESULTS OF DIFFERENT VISUALIZATION METHODS

This section shows how different methods visualize the single knee of the DEB4DK problem with $K = 1$.

The set of solutions approximating the Pareto front (also called approximation set) was obtained by sampling the front of the DEB4DK problem with 3000 points. Among them, eight solutions closest to the knee were chosen as the solutions "at the knee" to be emphasized in the visualizations (analogous to the red knee solutions emphasized in Figures 1 and 2). The underlying idea is that a visualization method should be capable of showing the knee solutions in a way that makes the knee recognizable to a decision maker.

Since some methods have difficulties when visualizing a large number of solutions, only the first 300 solutions were used in such cases. Out of the eight knee solutions contained in the original sample, five were retained in this smaller set.

3.1 Scatter Plot Matrix

Let us first view the results of the scatter plot matrix—a matrix of all possible 2-D projections of a multidimensional set of solutions. Figure 3 presents the scatter plot matrix for the smaller approximation set containing 300 solutions. Because of the projection to a 2-D space, most of the information is lost and the solutions at the knee cannot be differentiated from the rest of the solutions. Even an interactive exploration cannot be of much help in such a case.

3.2 Bubble chart

A bubble chart is a scatter plot in which three objectives are shown on axes while the fourth objective is represented with point size. Figure 4 contains the bubble chart for the

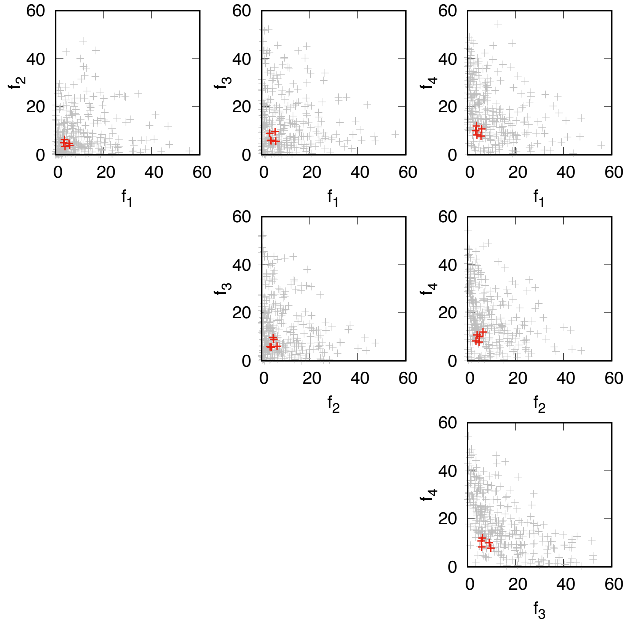

Figure 3: Scatter plot matrix of 300 Pareto-optimal solutions of the DEB4DK problem with a single knee (solutions near the knee are shown in red).

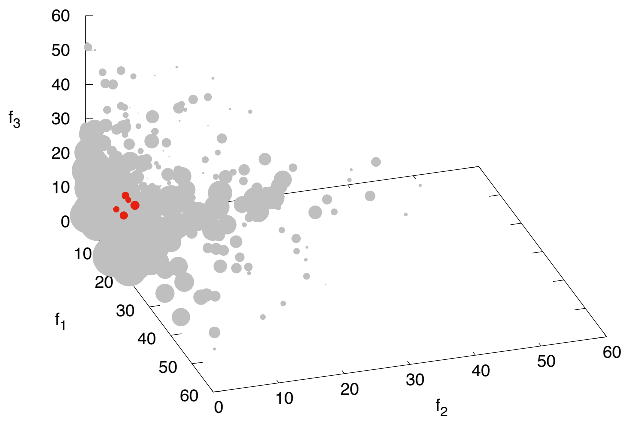

Figure 4: Bubble chart showing 300 Pareto-optimal solutions of the DEB4DK problem with a single knee (solutions near the knee are shown in red).

approximation set with 300 solutions. Similarly as before, this plot shows that the red knee solutions have low values in all four objectives, but we cannot perceive the presence of a knee.

3.3 Parallel Coordinates

Parallel coordinates [5] is a very popular method for visualizing multidimensional data that represents objectives as vertical parallel lines. Each solution is shown as a poly-line intersecting each objective (i.e. coordinate) at its value. In the case of our smaller approximation set (see Figure 5), the knee solutions do not stand out from the rest and therefore do not give a good idea regarding the shape of the approx-

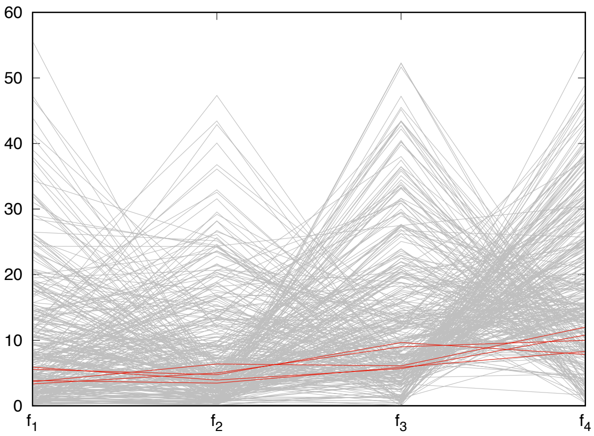

Figure 5: Parallel coordinates of 300 Pareto-optimal solutions of the DEB4DK problem with a single knee (solutions near the knee are shown in red).

imation set. Moreover, the method is very sensitive to the number of displayed solutions. When this number is small, it can be very informative and is able to show dominance relations between solutions. When, on the other hand, hundreds or thousands of solutions need to be visualized, the cluttered result causes loss of most of the information.

3.4 Radial Coordinate Visualization

The radial coordinate visualization (also called RadViz [4]) places objectives on the circumference of a unit circle and the solutions inside the circle so that the distance of a solution to each of the objectives is proportional to its value in that objective. For example, solutions with greater values in f_1 than in any other objective are placed closer to f_1 than the other objectives. Figure 6 shows the radial coordinate visualization of the entire approximation set with 3000 solutions. Again, the knee solutions cannot be differentiated from the rest in this plot.

3.5 Level Diagrams

Level diagrams [1] plot each solution against one of its objectives and the (Euclidean) distance to the ideal point. In case of four objectives, four such diagrams are produced. Figure 7 presents the level diagrams for the smaller approximation set and we can clearly see the knee in all of them.

3.6 Hyper-Radial Visualization

In hyper-radial visualization [3] the solutions are plotted against their distance to the ideal point (their *hyper-radius*) separately for two subsets of objectives. In our case, the x axis represents the hyper-radius for objectives f_1 and f_2, while the y axis represents the hyper-radius for objectives f_3 and f_4. As shown in Figure 8, the eight knee solutions from the larger approximation set are clearly recognizable in this visualization.

A note on the level diagrams and the hyper-radial visualization: while both methods are able to show the knee in this case, this is mostly due to the fact that the knee has the shortest distance to the ideal point. In case of multiple knees, for example, the knees that are not very close to

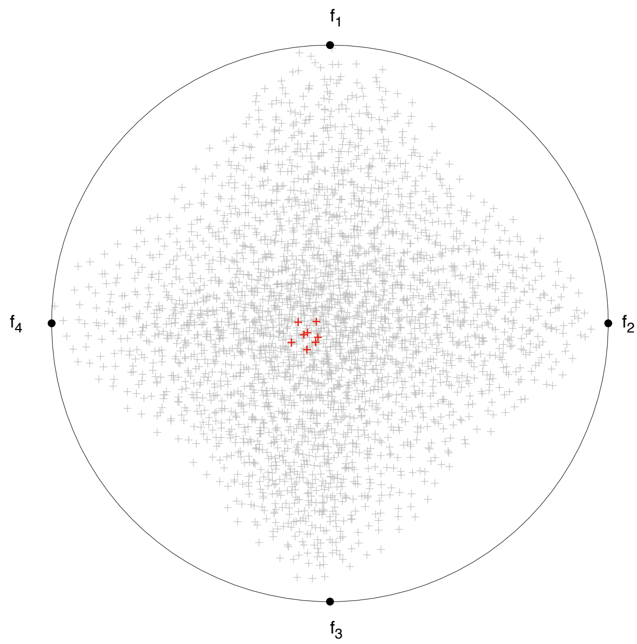

Figure 6: Radial coordinate visualization of 3000 Pareto-optimal solutions of the DEB4DK problem with a single knee (solutions near the knee are shown in red).

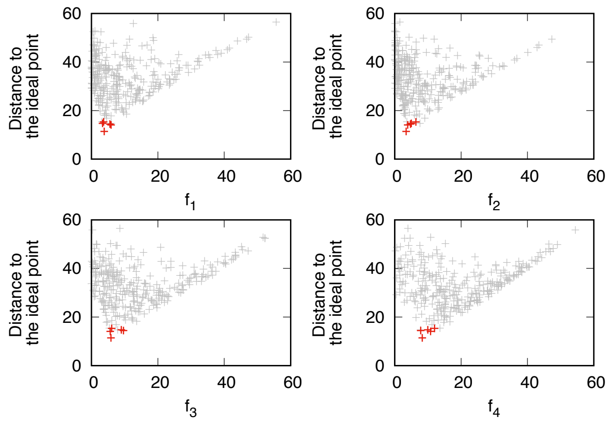

Figure 7: Level diagrams for 300 Pareto-optimal solutions of the DEB4DK problem with a single knee (solutions near the knee are shown in red).

the ideal point would probably be undetectable with these visualization methods. Showing this is left for future work.

3.7 Prosections

Prosections [6, 8] are projections of only a section of the objective space at a time. For example, a section of width d is selected at angle φ on the plane f_1f_2. All solutions that fall in this section are projected onto the line going through the ideal point at angle φ while all other solutions are discarded. By showing a 3-D scatter plot with the x axis representing the projected line and the other two axes

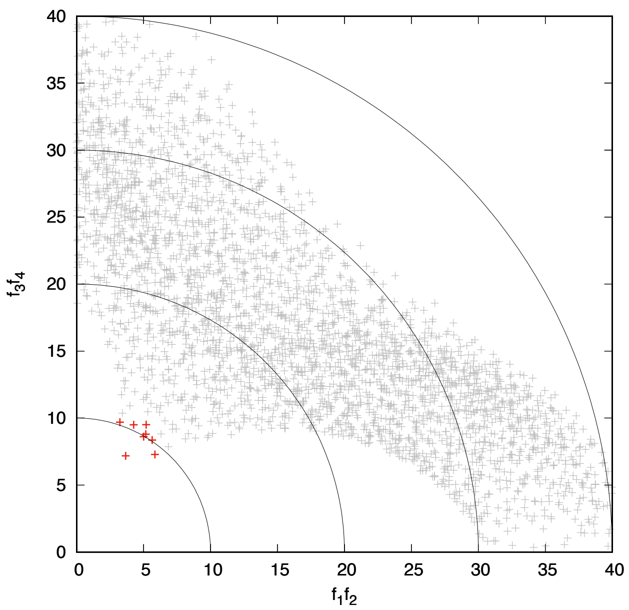

Figure 8: Hyper-radial visualization of 3000 Pareto-optimal solutions of the DEB4DK problem with a single knee (solutions near the knee are shown in red).

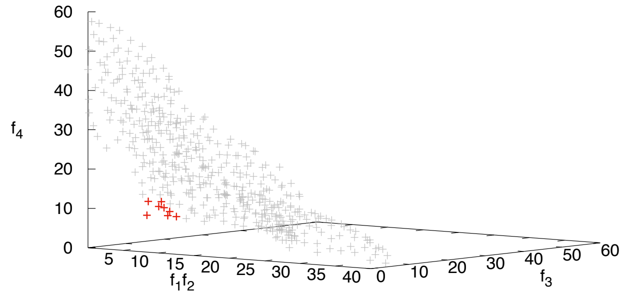

Figure 9: Prosection $4D(\mathbf{0}, f_1 f_2, 45°, 2)$ showing a part of 3000 Pareto-optimal solutions of the DEB4DK problem with a single knee (solutions near the knee are shown in red).

representing objectives f_3 and f_4, we get a 3-D representation of this section of the 4-D objective space that can be denoted as $4D(\mathbf{0}, f_1 f_2, \varphi, d)$. Figure 9 shows the prosection $4D(\mathbf{0}, f_1 f_2, 45°, 2)$, that is, a prosection on the plane $f_1 f_2$ using the angle $\varphi = 45°$ and section width $d = 2$. The knee points are clearly visible in this plot. In theory, prosections should be able to visualize any number of knees regardless of their position (using multiple plots—each visualizing a different section of the space), but experiments to demonstrate this are still needed.

The advantages and disadvantages of prosections originate from the fact that only a section of the space is visualized at a time. While this means that a visualization of high accuracy that mostly preserves the dominance relations among solutions is possible, it also requires to plot multiple visualizations (using various planes and angles) to get a complete view of an approximation set. Another advantage of this method is that it can easily handle large sets of solutions because only a part of the set is visualized in a single plot.

4. CONCLUSIONS

This paper presented how seven visualization methods previously used to visualize Pareto front approximations cope with visualization of knees. Only three methods (level diagrams, hyper-radial visualization and prosections) were able to show the single knee of the four-objective DEB4DK problem. Additional work is needed to see how these visualization methods perform on a problem with more knees.

5. ACKNOWLEDGMENTS

This work is part of a project that has received funding from the *European Union's Horizon 2020 research and innovation program* under grant agreement No. 692286. This work was partially funded by the Slovenian Research Agency under research program P2-0209.

6. REFERENCES

[1] X. Blasco, J. M. Herrero, J. Sanchis, and M. Martínez. A new graphical visualization of n-dimensional Pareto front for decision-making in multiobjective optimization. *Information Sciences*, 178(20):3908–3924, 2008.

[2] J. Branke, K. Deb, H. Dierolf, and M. Oswald. Finding knees in multi-objective optimization. In *Proceedings of the 8th International Conference on Parallel Problem Solving from Nature, PPSN VIII, Birmingham, UK*, volume 3242 of *Lecture Notes in Computer Science*, pages 722–731. Springer, 2004.

[3] P.-W. Chiu and C. Bloebaum. Hyper-radial visualization (HRV) method with range-based preferences for multi-objective decision making. *Structural and Multidisciplinary Optimization*, 40(1–6):97–115, 2010.

[4] P. E. Hoffman, G. G. Grinstein, K. Marx, I. Grosse, and E. Stanley. DNA visual and analytic data mining. In *Proceedings of the IEEE Conference on Visualization*, pages 437–441, Los Alamitos, CA, USA, 1997. IEEE.

[5] A. Inselberg. *Parallel Coordinates: Visual Multidimensional Geometry and its Applications.* Springer, New York, NY, USA, 2009.

[6] T. Tušar and B. Filipič. Visualizing 4D approximation sets of multiobjective optimizers with prosections. In *Proceedings of the Genetic and Evolutionary Computation Conference, GECCO '11, Dublin, Ireland*, pages 737–744. ACM, 2011.

[7] T. Tušar and B. Filipič. Scaling and visualizing multiobjective optimization test problems with knees. In *Proceedings of the 15th International Multiconference Information Society, IS 2012, Ljubljana, Slovenia*, volume A, pages 155–158, Ljubljana, Slovenia, 2012. Jožef Stefan Institute.

[8] T. Tušar and B. Filipič. Visualization of Pareto front approximations in evolutionary multiobjective optimization: A critical review and the prosection method. *IEEE Transactions on Evolutionary Computation*, 19(2):225–245, 2015.