EMO 2023 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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The final slides will be made available at
http://www.cmap.polytechnique.fr/~dimo.brockhoff/

Overview

Our plan

Discuss history, present and future of multiobjective benchmarking

With respect to different topics

- performance assessment / methodology
- test functions

Finally, recommendations on good algorithms

Disclaimer

We only consider continuous search spaces

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We only consider unconstrained problems

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What we present is highly subjective & selective

- how important do we find each milestone?
- use version numbering and branches
- what have we learned from the past?

Overview

❶ Performance Assessment

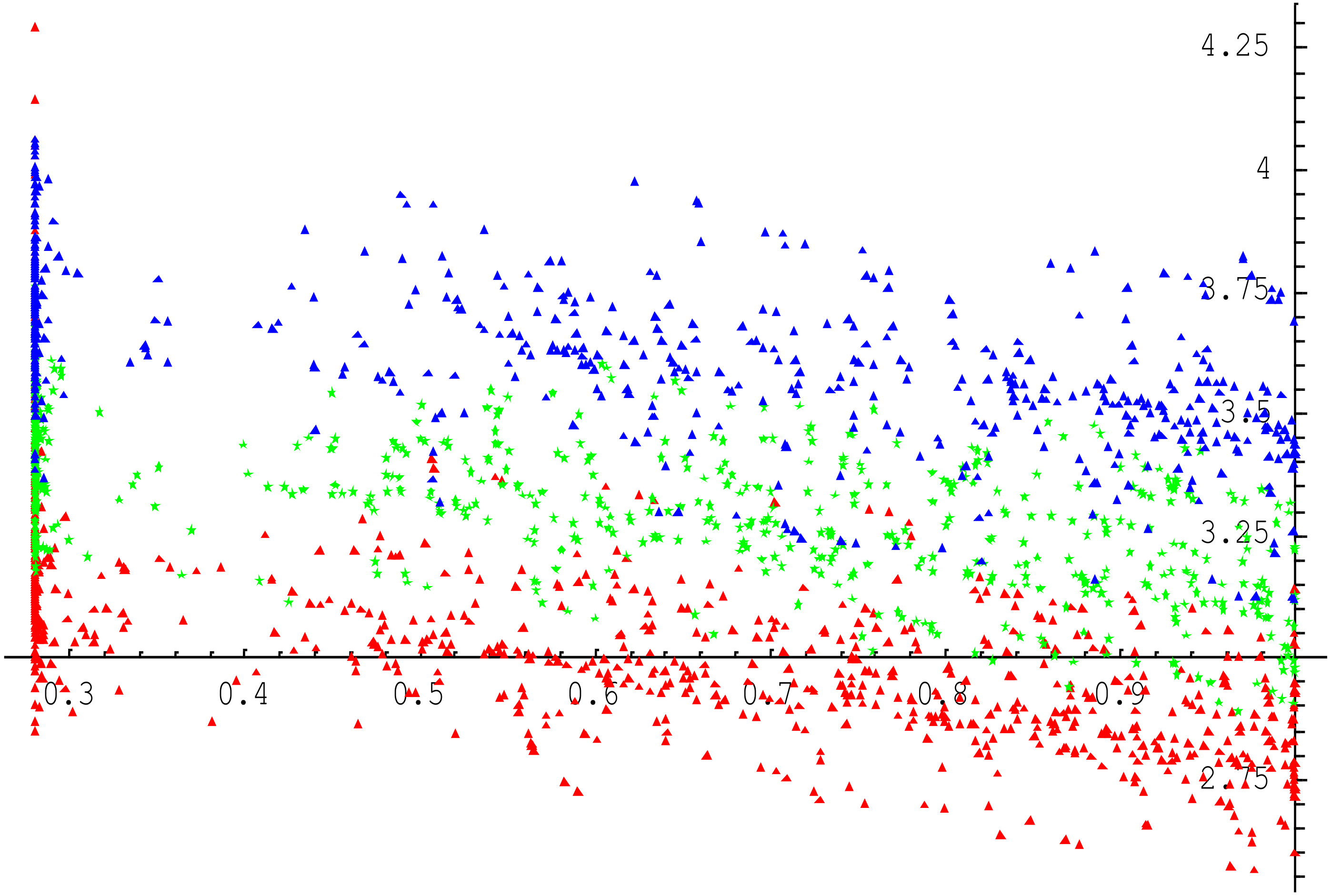
❷ Test Problems and Their Visualizations

❸ Recommendations from Numerical Results

v0.0.1alpha

In The Early Beginnings…

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

v0.1beta

Tables

Problem	MOCSA				NSGA2			
	$\langle d \rangle$	S	GD	ER	$\langle d \rangle$	S	GD	ER
ZDT1	0.0404	0.0055	0.0000	0	0.0270	0.0156	0.0011	0.04
ZDT2	0.0404	0.0082	0.0000	0	0.0292	0.0146	0.0212	0.02
ZDT3	0.0438	0.0148	0.0001	0	0.0329	0.0201	0.0020	0.02
ZDT4	0.0404	0.0097	0.0000	0	0.0328	0.0159	0.0006	0.02
ZDT6	0.0327	0.0150	0.0000	0	0.0216	0.0119	0.0000	0
DTLZ1	0.1114	0.0068	0.0000	0	0.0615	0.0319	0.0000	0
DTLZ2	0.2319	0.0646	0.0021	0.02	0.1361	0.0683	0.0020	0.04
DTLZ3	0.2770	0.0225	0.0000	0	0.1139	0.0739	0.0000	0
DTLZ4	0.2478	0.0424	0.0009	0	0.1630	0.0898	0.0019	0.02
DTLZ5	0.0487	0.0059	0.0000	0	0.0309	0.0176	0.0610	0.06
DTLZ6	0.0484	0.0156	0.0000	0	0.0306	0.0135	0.0000	0
DTLZ7	0.2897	0.0510	0.0011	0.04	0.1880	0.1322	0.0071	0.22

arXiv, 2012

Tables

Table 4: Influence of different κ values on the performance of the coneϵ-MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for κ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Metric		κ ; Deb52										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.0001	0.0001
Δ	M	0.6766	0.6813	0.5244	0.2991	0.2552	0.2432	0.2648	0.2892	0.3147	0.3194	0.3199
	SD	0.0004	0.0021	0.0025	0.0027	0.0034	0.0039	0.0017	0.0019	0.0016	0.0042	0.0066
HV	M	0.2735	0.2779	0.2794	0.2802	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
$\lvert\mathcal{H}\rvert$	M	19.00	51.00	74.00	93.00	101.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	$< 10^{-4}$	$< 10^{-4}$	0.2537	0.3457	0.4842	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.1826	0.1826	0.1826

Metric		κ ; ZDT1										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0103	0.0069	0.0055	0.0059	0.0074	0.0040	0.0042	0.0051	0.0053	0.0050	0.0038
	SD	0.0072	0.0038	0.0057	0.0047	0.0049	0.0042	0.0060	0.0058	0.0040	0.0050	0.0034
Δ	M	0.3046	0.5543	0.3678	0.2084	0.1818	0.1812	0.1898	0.1937	0.1934	0.1956	0.1891
	SD	0.0122	0.0607	0.0480	0.0408	0.0235	0.0220	0.0234	0.0251	0.0240	0.0232	0.0155
HV	M	0.8435	0.8561	0.8602	0.8607	0.8598	0.8652	0.8650	0.8636	0.8633	0.8638	0.8657
	SD	0.0115	0.0066	0.0094	0.0079	0.0082	0.0069	0.0099	0.0096	0.0066	0.0083	0.0057
$\lvert\mathcal{H}\rvert$	M	37.00	63.00	84.50	98.00	100.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	0.6397	5.7211	2.8730	5.0901	3.8201	0.5467	0.8584	0.9371	0.9377	1.3515	0.7112

Metric		κ ; DTLZ2 (m = 3)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0062	0.0069	0.0072	0.0070	0.0074	0.0079	0.0074	0.0076	0.0074	0.0078	0.0072
	SD	0.0002	0.0013	0.0015	0.0013	0.0012	0.0014	0.0010	0.0019	0.0007	0.0014	0.0009
Δ	M	0.0503	0.6066	0.3029	0.2411	0.2386	0.2308	0.2274	0.2175	0.2079	0.2173	0.1982
	SD	0.0041	0.0422	0.0357	0.0302	0.0264	0.0219	0.0316	0.0275	0.0306	0.0295	0.0239
HV	M	0.6731	0.7149	0.7383	0.7435	0.7458	0.7469	0.7467	0.7469	0.7470	0.7470	0.7471
	SD	0.0066	0.0042	0.0023	0.0012	0.0007	0.0006	0.0005	0.0005	0.0005	0.0005	0.0003
$\lvert\mathcal{H}\rvert$	M	21.00	69.00	88.00	93.00	94.50	95.00	95.00	95.00	95.00	95.00	94.00
	SD	1.3047	3.1639	2.8367	2.0424	1.7750	1.9464	2.2894	2.0197	1.5643	2.2614	1.7100

Metric		κ ; DTLZ2 (m = 4)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0001	0.0311	0.0385	0.0312	0.0449	0.0404	0.0445	0.0488	0.0590	0.0489	0.0534
	SD	0.0001	0.0198	0.0218	0.0239	0.0283	0.0240	0.0369	0.0281	0.0284	0.0241	0.0304
Δ	M	0.1390	0.4700	0.3602	0.3296	0.3299	0.3429	0.3377	0.3258	0.3253	0.3304	0.3319
	SD	0.1173	0.0304	0.0307	0.0226	0.0255	0.0263	0.0187	0.0262	0.0210	0.0254	0.0259
$\lvert\mathcal{H}\rvert$	M	14.00	79.50	90.00	92.00	95.00	96.00	95.50	97.00	98.00	95.50	97.00
	SD	1.9815	4.9642	4.8476	4.2372	4.6307	4.2129	5.8530	5.0496	4.5945	4.6233	4.3423

arXiv, 2020

Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ-MOEA	coneϵ-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Deb52	γ	0.58±7e-3	0.56±3e-3	0.56±2e-3	0.58±1e-2	0.55±7e-3	0.55±6e-3
	Δ	0.53±8e-3	0.99±6e-4	0.32±3e-4	0.33±6e-3	0.20±3e-3	0.41±8e-3
	HV	0.99±7e-5	0.99±6e-5	0.99±0	0.99±1e-4	0.99±3e-5	0.99±7e-5
	CS	0.02±7e-4	0.03±10e-4	0.03±8e-4	0.02±8e-4	0.03±9e-4	0.02±8e-4
Pol	γ	0.20±2e-2	0.13±6e-4	0.19±2e-2	0.19±9e-3	0.15±2e-3	0.16±2e-3
	Δ	0.58±1e-2	0.98±9e-4	0.29±6e-3	0.38±7e-3	0.24±3e-3	0.36±8e-3
	HV	1.00±1e-5	1.00±5e-6	1.00±4e-6	1.00±3e-5	1.00±4e-6	1.00±8e-6
	CS	0.04±1e-3	0.04±1e-3	0.04±2e-3	0.04±10e-4	0.06±1e-3	0.05±1e-3
Zdt1	γ	0.16±2e-2	0.30±3e-2	0.23±3e-2	0.18±2e-2	0.30±2e-2	0.19±2e-2
	Δ	0.79±1e-2	0.70±4e-3	0.37±6e-3	0.50±8e-3	0.29±7e-3	0.56±1e-2
	HV	0.99±10e-4	0.98±2e-3	0.98±2e-3	0.98±9e-4	0.98±1e-3	0.98±1e-3
	CS	0.33±2e-2	0.20±3e-2	0.27±3e-2	0.30±2e-2	0.15±2e-2	0.27±2e-2
Zdt2	γ	0.43±9e-3	0.65±8e-3	0.30±4e-3	0.80±1e-2	0.41±8e-3	0.41±8e-3
	Δ	0.76±1e-2	0.56±3e-3	0.38±4e-3	0.50±7e-3	0.28±4e-3	0.58±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.99±4e-5	0.98±1e-4	0.99±8e-5	0.99±9e-5
	CS	0.07±3e-3	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Zdt3	γ	0.16±2e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Δ	0.67±1e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	HV	0.98±2e-3	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	CS	0.41±3e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Zdt4	γ	0.27±2e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Δ	0.61±9e-3	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	HV	0.84±9e-3	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	CS	0.73±3e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Zdt6	γ	0.06±3e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Δ	0.52±1e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	HV	0.96±1e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	CS	0.18±2e-2	0.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ-MOEA	coneϵ-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Dtlz1	γ	0.23±7e-3	0.13±1e-3	0.17±2e-2	0.39±1e-2	0.18±2e-3	0.17±2e-3
	Δ	0.34±3e-3	0.12±2e-3	0.05±1e-2	0.20±2e-2	0.08±1e-3	0.34±4e-3
	HV	0.95±6e-4	0.92±4e-4	0.95±2e-4	0.96±5e-4	0.97±1e-4	0.96±5e-4
	CS	0.02±8e-4	0.01±9e-4	0.03±1e-3	0.00±5e-4	0.02±1e-3	0.02±10e-4
Dtlz2	γ	0.62±10e-3	0.70±7e-3	0.48±8e-3	0.75±1e-2	0.55±8e-3	0.48±6e-3
	Δ	0.81±10e-3	0.42±4e-3	0.42±6e-3	0.29±5e-3	0.16±3e-3	0.83±9e-3
	HV	0.89±9e-4	0.92±4e-4	0.94±1e-4	0.90±7e-4	0.93±4e-4	0.89±9e-4
	CS	0.03±1e-3	0.02±8e-4	0.06±2e-3	0.01±6e-4	0.03±1e-3	0.04±1e-3
Dtlz3	γ	0.35±2e-2	0.25±1e-2	0.42±3e-2	0.50±3e-2	0.32±2e-2	0.26±1e-2
	Δ	0.33±9e-3	0.19±1e-2	0.29±2e-2	0.22±3e-2	0.15±2e-2	0.34±8e-3
	HV	0.90±10e-4	0.91±2e-2	0.91±1e-2	0.91±1e-3	0.93±3e-4	0.90±9e-4
	CS	0.02±1e-3	0.03±2e-3	0.04±2e-3	0.00±5e-4	0.02±2e-3	0.02±2e-3
Dtlz4	γ	0.32±8e-3	0.41±2e-2	0.53±3e-2	0.34±1e-2	0.33±1e-2	0.30±3e-3
	Δ	0.67±2e-2	0.37±3e-2	0.43±2e-2	0.36±4e-2	0.22±3e-2	0.66±8e-3
	HV	0.88±1e-2	0.86±2e-2	0.86±2e-2	0.84±2e-2	0.87±2e-2	0.90±7e-4
	CS	0.04±2e-3	0.02±1e-3	0.03±2e-3	0.02±2e-3	0.03±2e-3	0.03±1e-3
Dtlz5	γ	0.14±2e-3	0.26±3e-3	0.56±3e-2	0.22±5e-3	0.15±2e-3	0.13±1e-3
	Δ	0.74±2e-2	0.78±5e-3	0.83±9e-3	0.43±6e-3	0.26±4e-3	0.61±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.98±3e-5	0.98±1e-4	0.99±4e-5	0.99±1e-4
	CS	0.06±2e-3	0.02±1e-3	0.04±1e-3	0.03±1e-3	0.05±2e-3	0.07±2e-3
[illegible]	γ	0.84±8e-3	0.94±3e-3	0.83±3e-3	0.83±5e-3	0.84±7e-3	0.84±8e-3
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]1±7e-3	0.32±5e-3	0.62±1e-2
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]±9e-5	0.99±3e-5	0.99±1e-4
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]8e-4	0.03±8e-4	0.01±7e-4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]9e-3	0.67±8e-3	0.74±10e-3
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]1e-2	0.52±8e-3	0.78±8e-3
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]1e-3	0.94±6e-4	0.92±9e-4
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]1e-3	0.09±3e-3	0.07±2e-3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]1e-2	0.83±9e-3	0.58±1e-2
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]8e-3	0.36±8e-3	0.80±1e-2
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]3e-3	0.10±3e-3	0.09±3e-3
Dtlz9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]4±7e-3	0.77±7e-3	0.90±5e-3
	Δ	0.62±5e-3	0.71±1e-2	0.52±6e-3	0.49±6e-3	0.45±4e-3	0.61±6e-3
	CS	0.11±8e-3	0.51±1e-2	0.34±7e-3	0.23±8e-3	0.24±8e-3	0.13±7e-3

Numbers have their value.

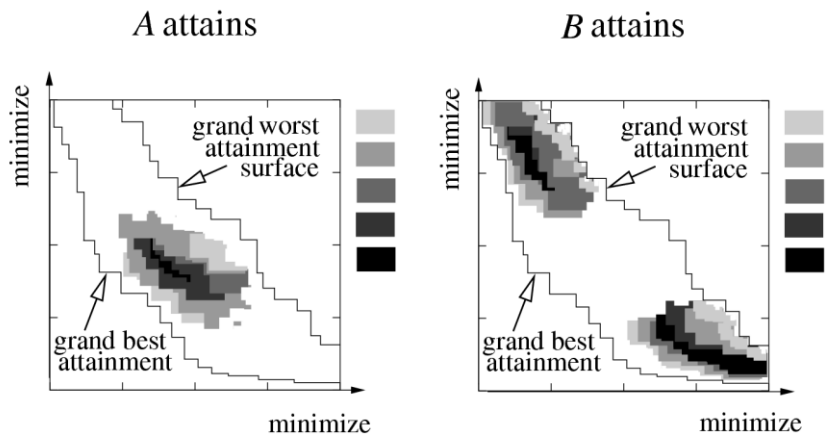
But not *only* tables, please!

v1.0

v1.0: Two Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur



Quality indicator approach

- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions

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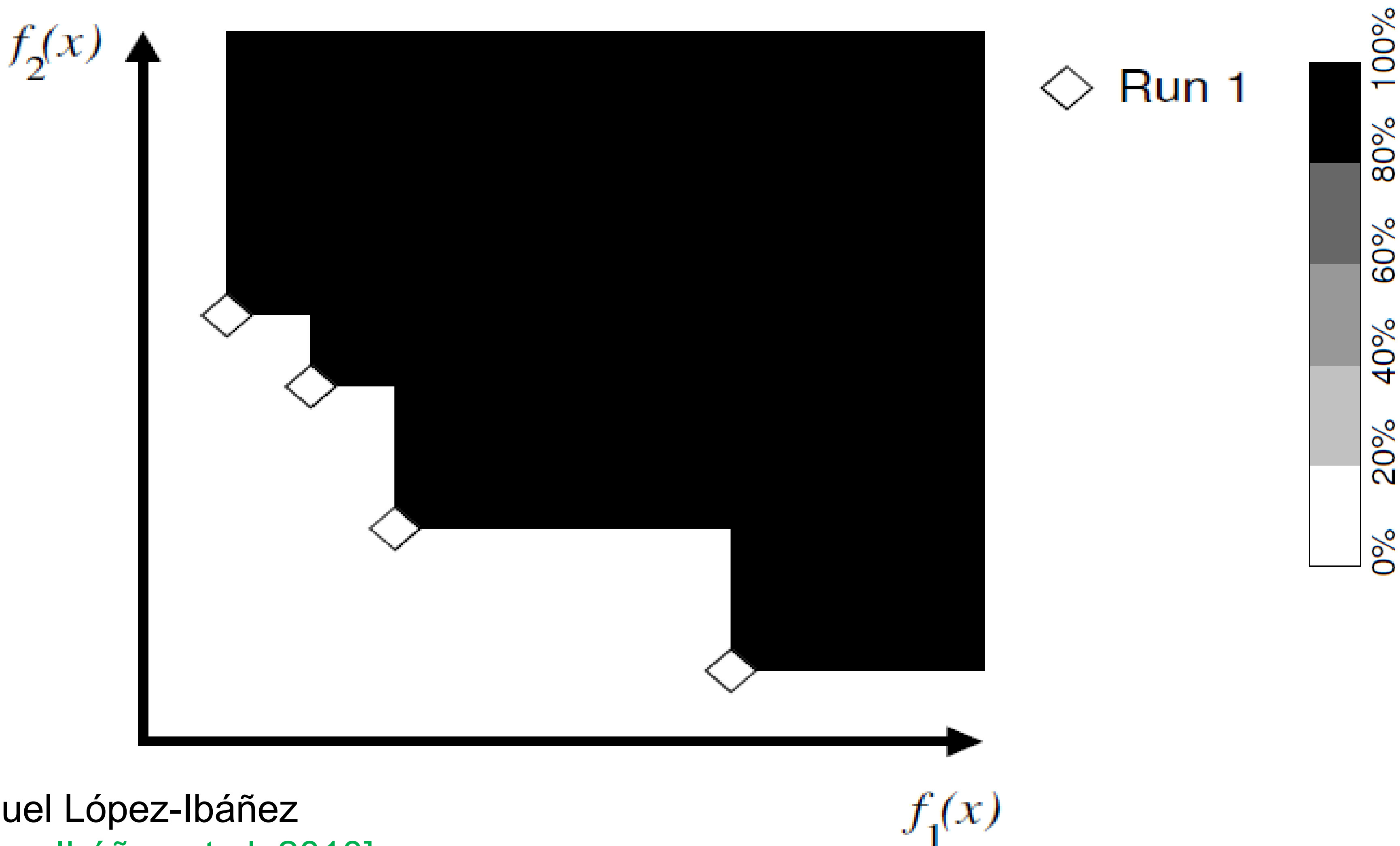
[López-Ibáñez et al. 2010]

Empirical Attainment Functions



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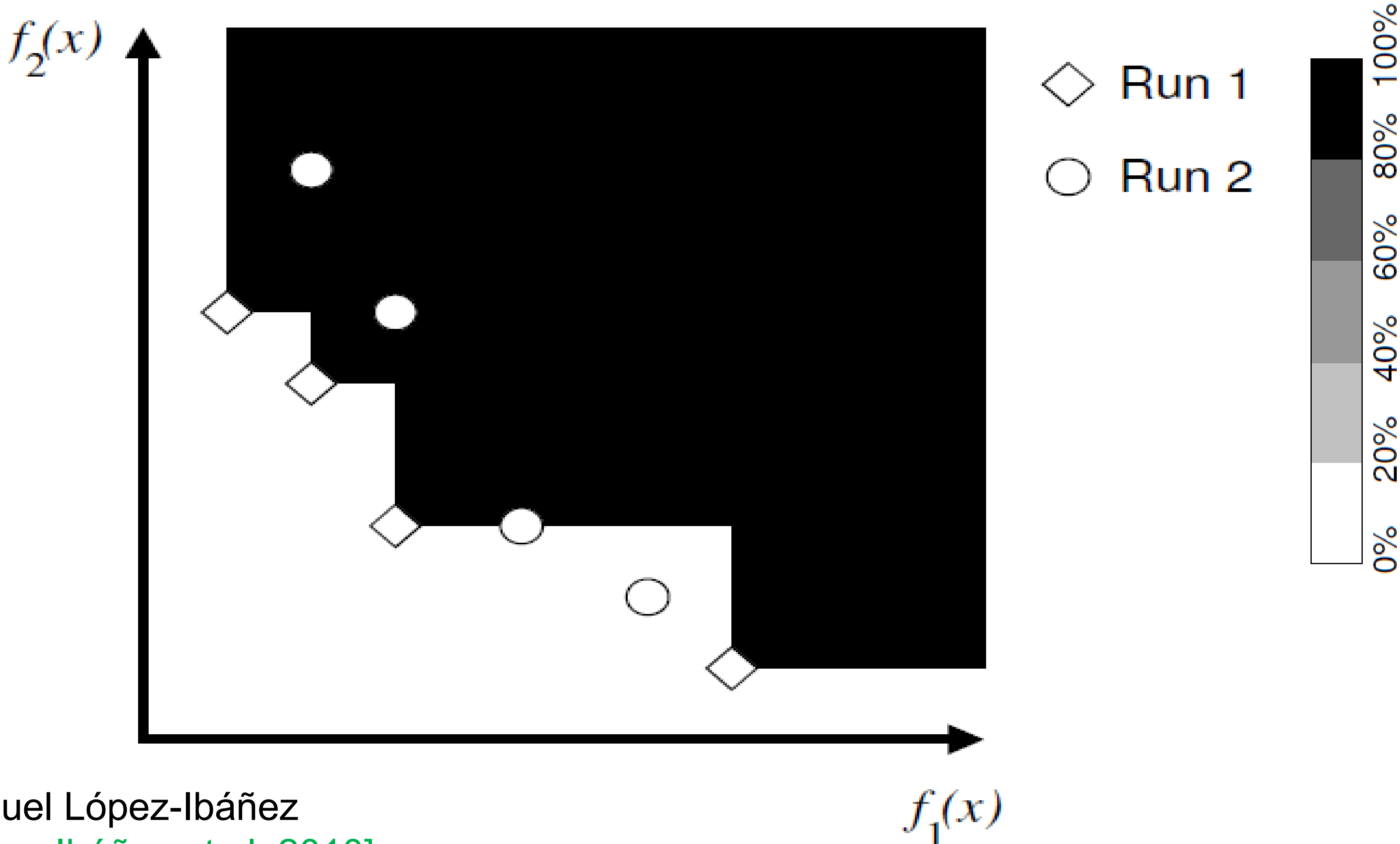
Empirical Attainment Functions



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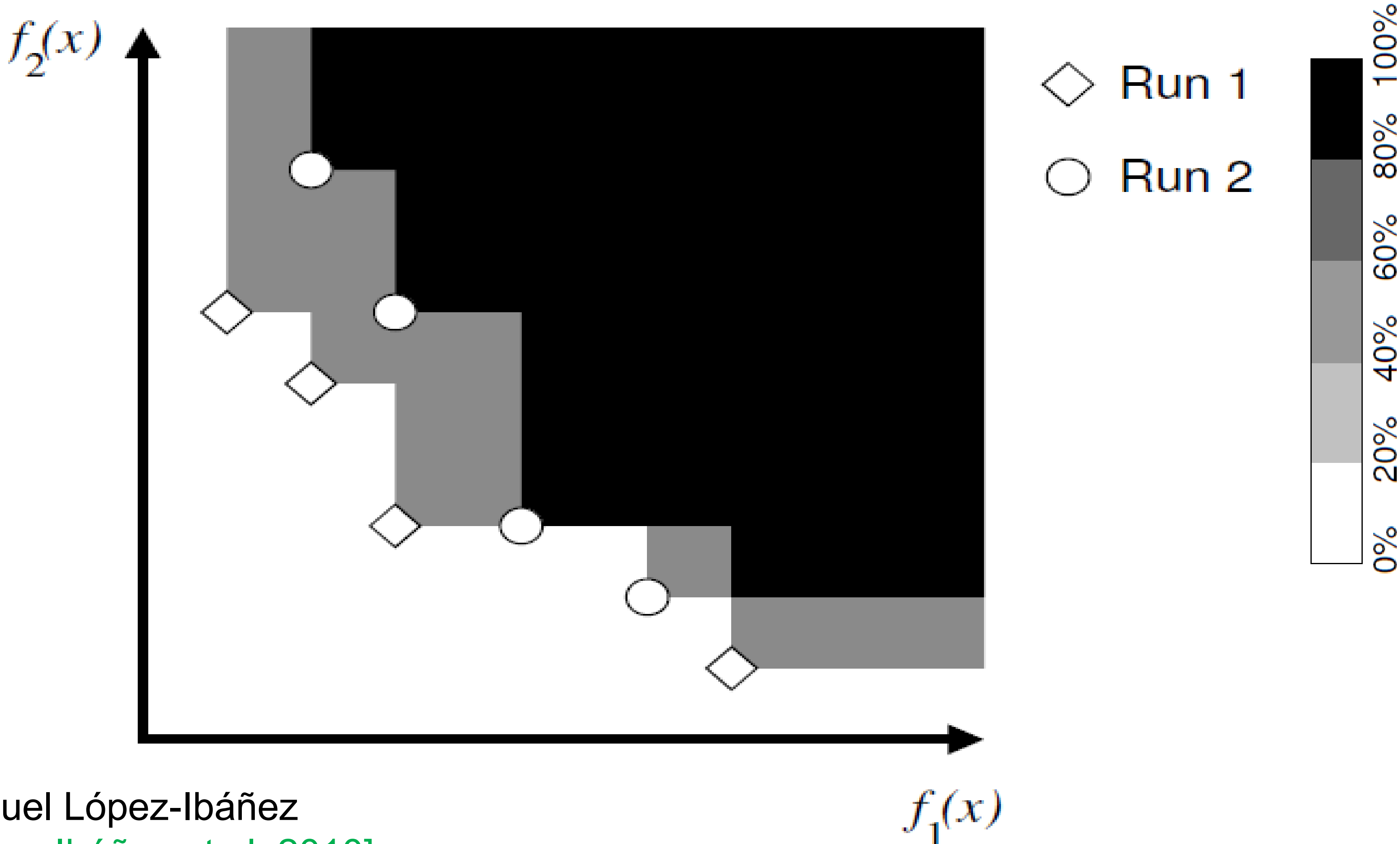
Empirical Attainment Functions



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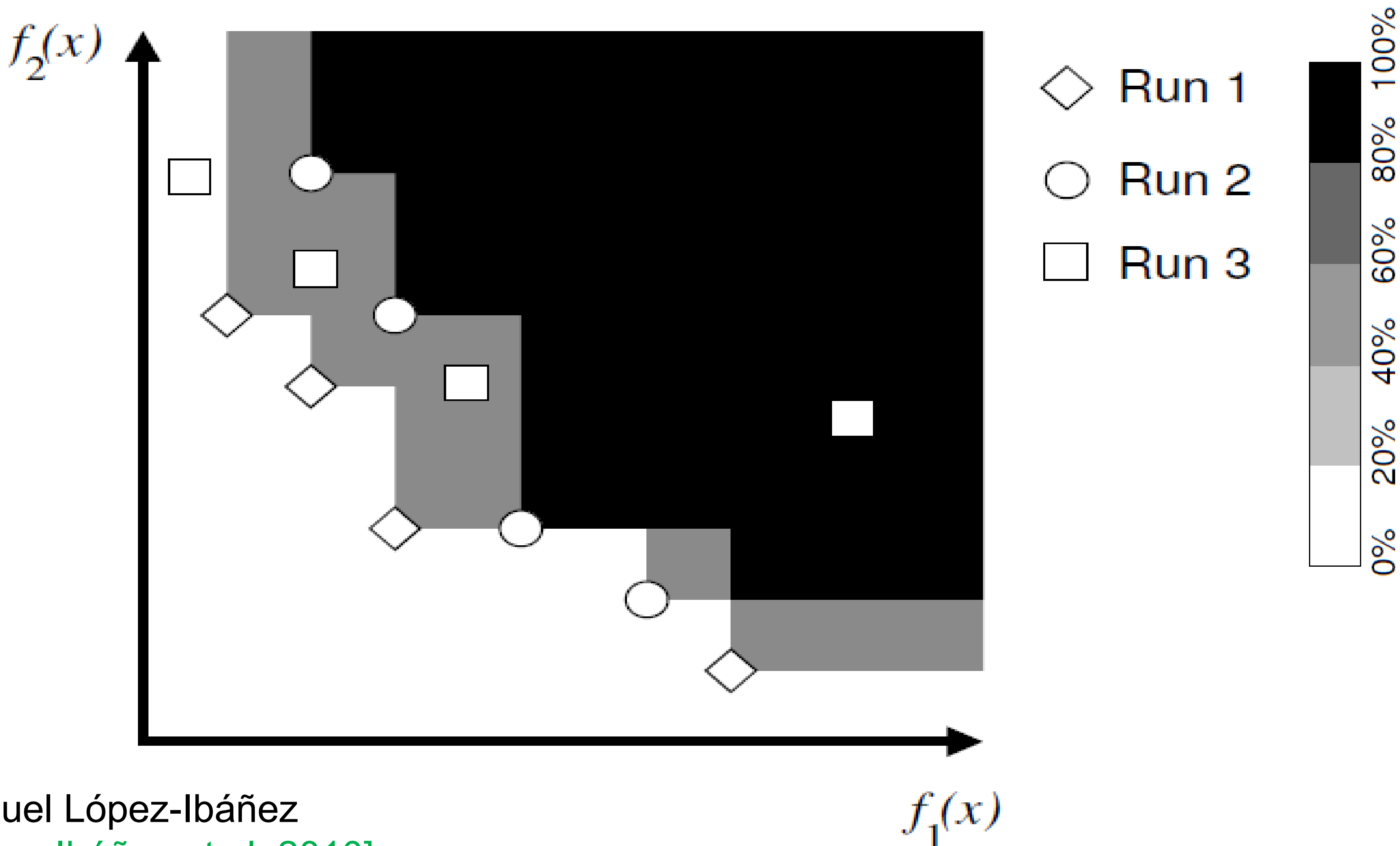
Empirical Attainment Functions



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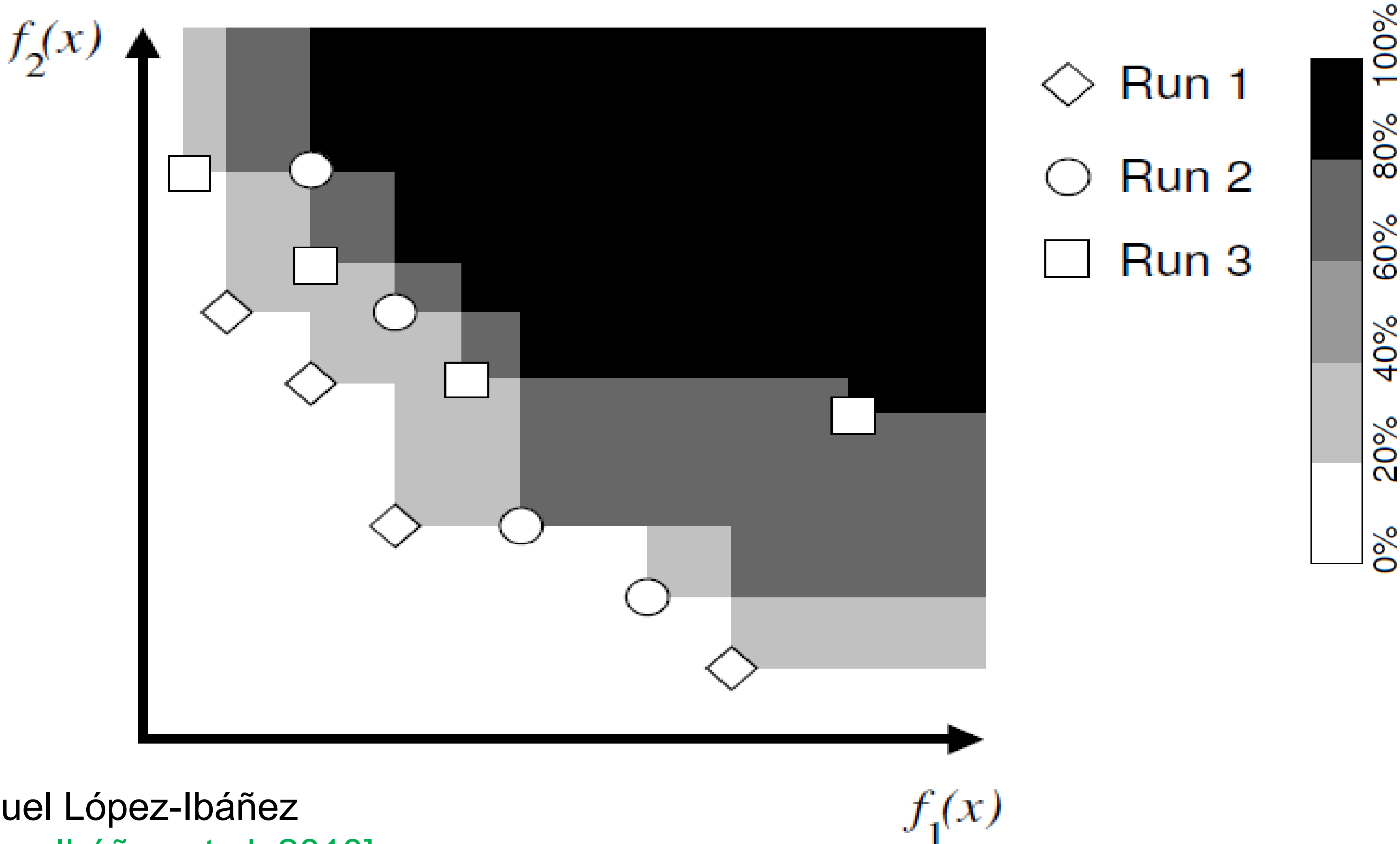
Empirical Attainment Functions



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[López-Ibáñez et al. 2010]

Empirical Attainment Functions: Definition

The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector z at time T:

$$\alpha_T(z) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\{\mathcal{X}_i \trianglelefteq_T z\}}$$

with $\trianglelefteq_T$ being the weak dominance relation between a solution set and an objective vector at time T.

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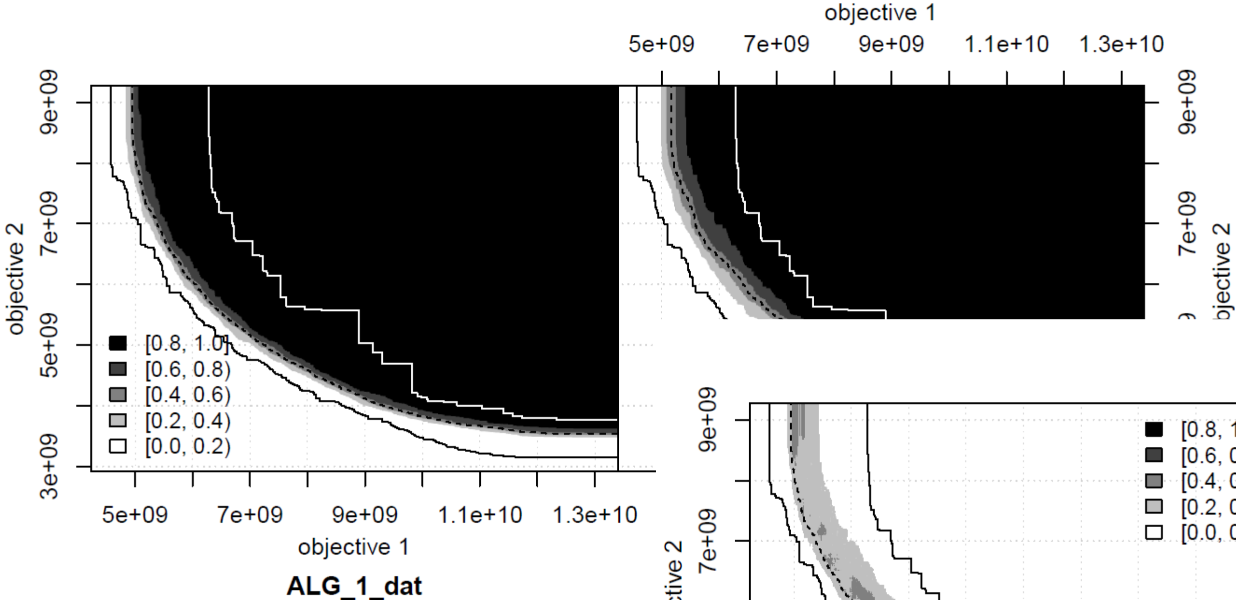
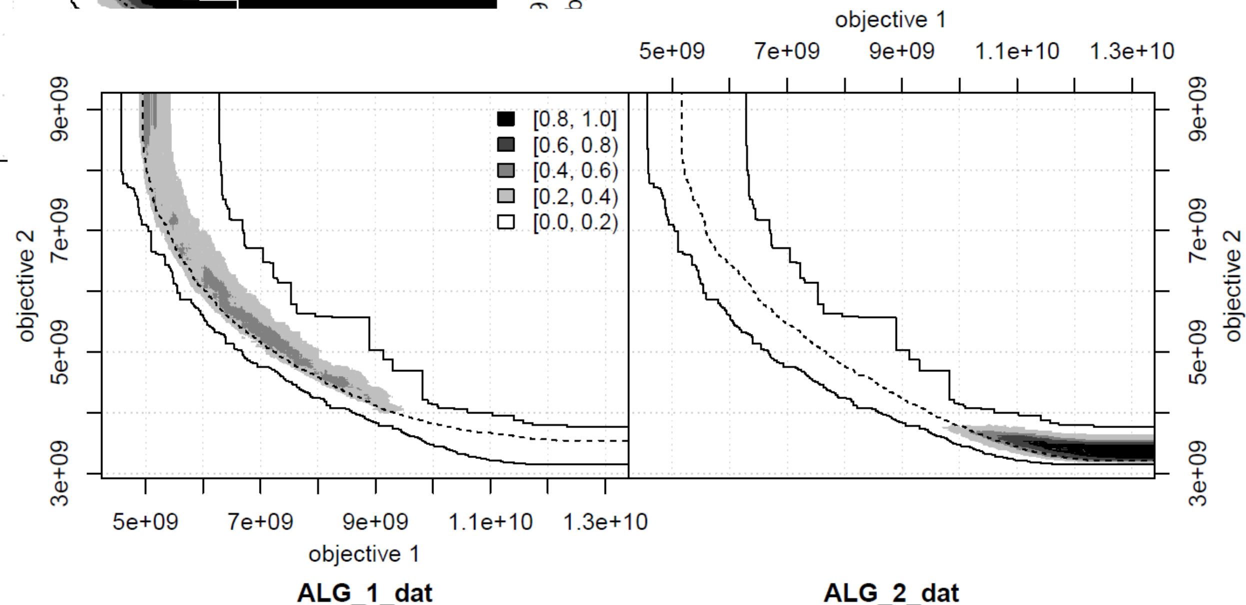
Note that $\alpha_T(z)$ is the empirical cumulative distribution function of the achieved objective function distribution at time T in the single-objective case ("fixed budget scenario").

Empirical Attainment Functions in Practice





© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
R package: **`http://lopez-ibanez.eu/eaftools`**
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function (“unary quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

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Attention:

Underlying dominance relation should be reflected!

$$A \preccurlyeq B :\Leftrightarrow \forall_{b \in B} \exists_{a \in A}\, a \preccurlyeq b$$

Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation

$$A \preccurlyeq B \Rightarrow I(A) \geq I(B)$$

Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation

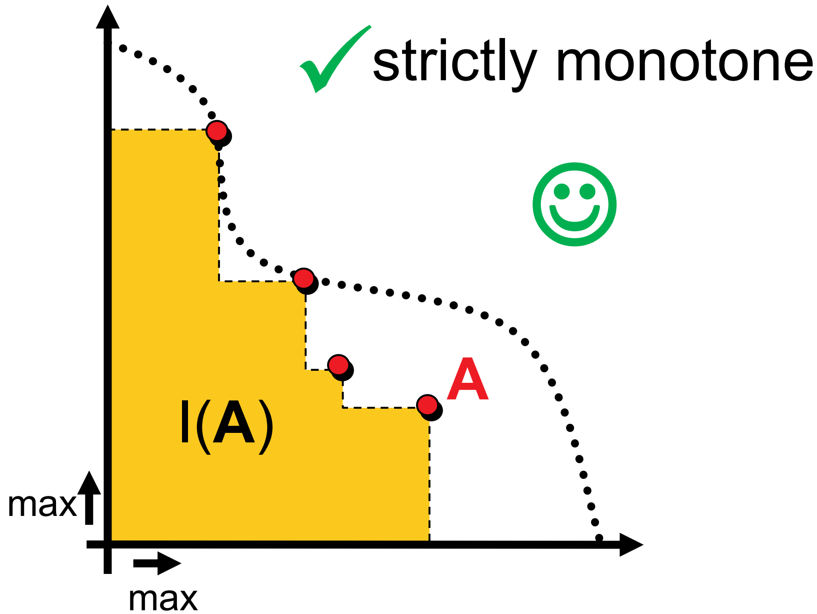
$$A \preccurlyeq B \Rightarrow I(A) \geq I(B)$$

- Strict monotonicity: better = higher indicator

$$A \preccurlyeq B \text{ and } A \neq B \Rightarrow I(A) > I(B)$$

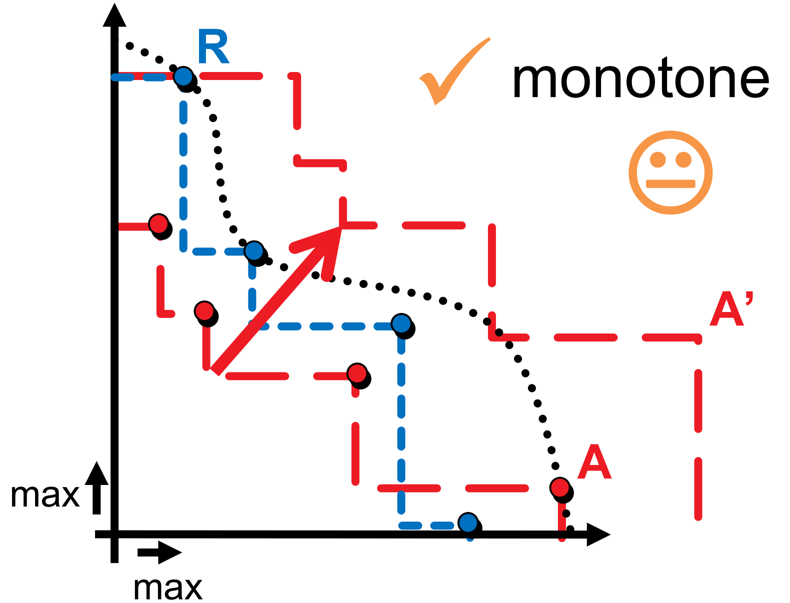
Example: Refinements Using Indicators

I(A) = volume of the weakly dominated area in objective space



unary hypervolume indicator

I(A,R) = how much needs A to be moved to weakly dominate R



unary epsilon indicator

v1.0.1 – v1.0.100 and counting

Many Indicators Available

Performance Assessment of Multiobjective Optimizers: An Analysis and Review

Eckart Zitzler[1], Lothar Thiele[1], Marco Laumanns[1],
Carlos M. Fonseca[2], and Viviane Grunert da Fonseca[2]

[1] Computer Engineering and Networks Laboratory (TIK)
Department of Information Technology and Electrical Engineering
Swiss Federal Institute of Technology (ETH) Zurich, Switzerland
Email: {zitzler, thiele, laumanns}@tik.ee.ethz.ch

[2]ADEEC and ISR (Coimbra)
Faculty of Sciences and Technology
University of Algarve, Portugal
Email: cmfonsec@ualg.pt, vgrunert@csi.fct.ualg.pt

[Zitzler et al. 2003]

22 indicators

Even More Indicators…

Performance indicators in multiobjective optimization

Charles Audet[a], Jean Bigeon[b], Dominique Cartier[c], Sébastien Le Digabel[a], Ludovic Salomon[a,1]

[a] *GERAD and Département de mathématiques et génie industriel, École Polytechnique de Montréal, C.P. 6079, Succ. Centre-ville, Montréal, Québec, H3C 3A7, Canada.*
[b] *Univ. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, 38000 Grenoble, France.*
[c] *Collège de Maisonneuve, 3800 Rue Sherbrooke E, Montréal, Québec, H1X 2A2, Canada.*

[Audet et al 2021]

63 indicators

Quality Evaluation of Solution Sets in Multiobjective Optimisation: A Survey

Miqing Li, and Xin Yao[1]

[1] CERCIA, School of Computer Science, University of Birmingham, Birmingham B15 2TT, U. K.
*Email: limitsing@gmail.com, x.yao@cs.bham.ac.uk

[Li and Yao 2019]

100 indicators

Many Indicators: What Do We Do?

Focus on indicators which are (strictly) monotone

- all hypervolume-based indicators
- unary epsilon indicator
- R2
- IGD+

v2.0

Benchmarking Multiobjective Optimizers 2.0

- With the right (strictly) monotone indicator, multiobjective optimization is not different from single-objective optimization (!)

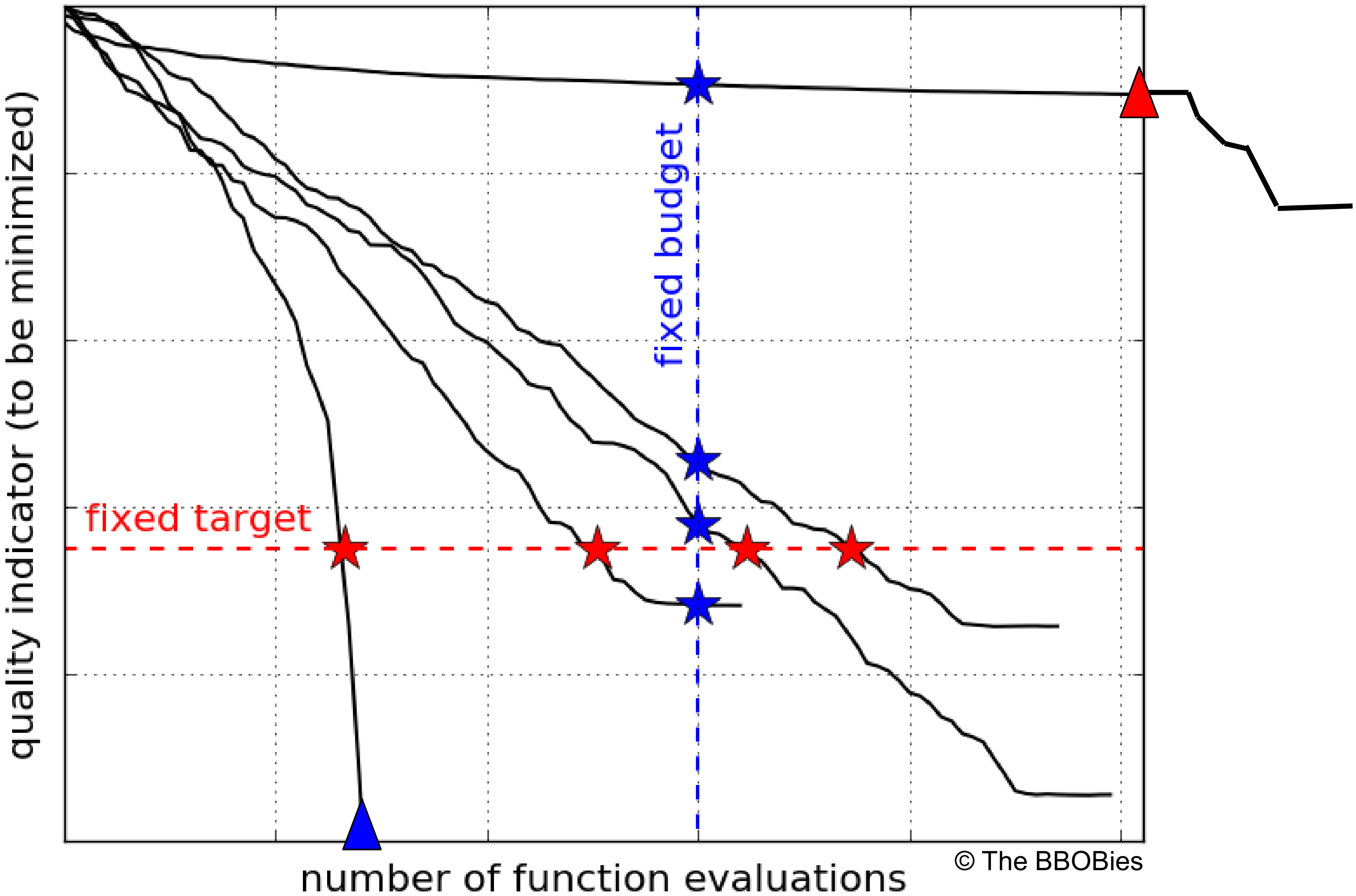
We can use our normal tools from single-objective optimization, including

- reporting of target-based runtimes
- ECDFs of runtimes, performance profiles, data profiles
- statistical tests, box plots, …

see for example [Hansen et al. 2021]

Measuring Performance Empirically

convergence graphs is all we have to start with...



Measuring Performance Empirically

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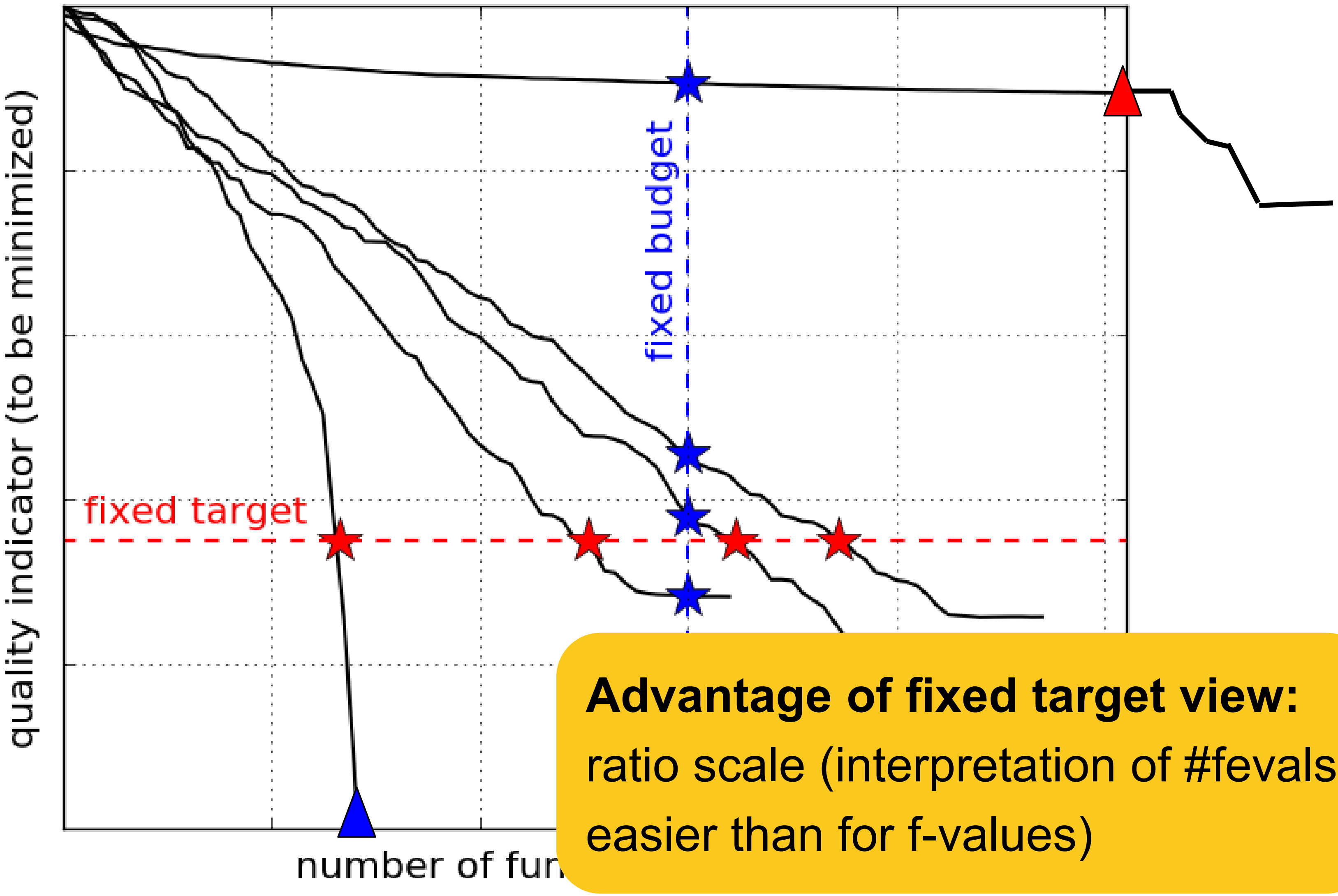
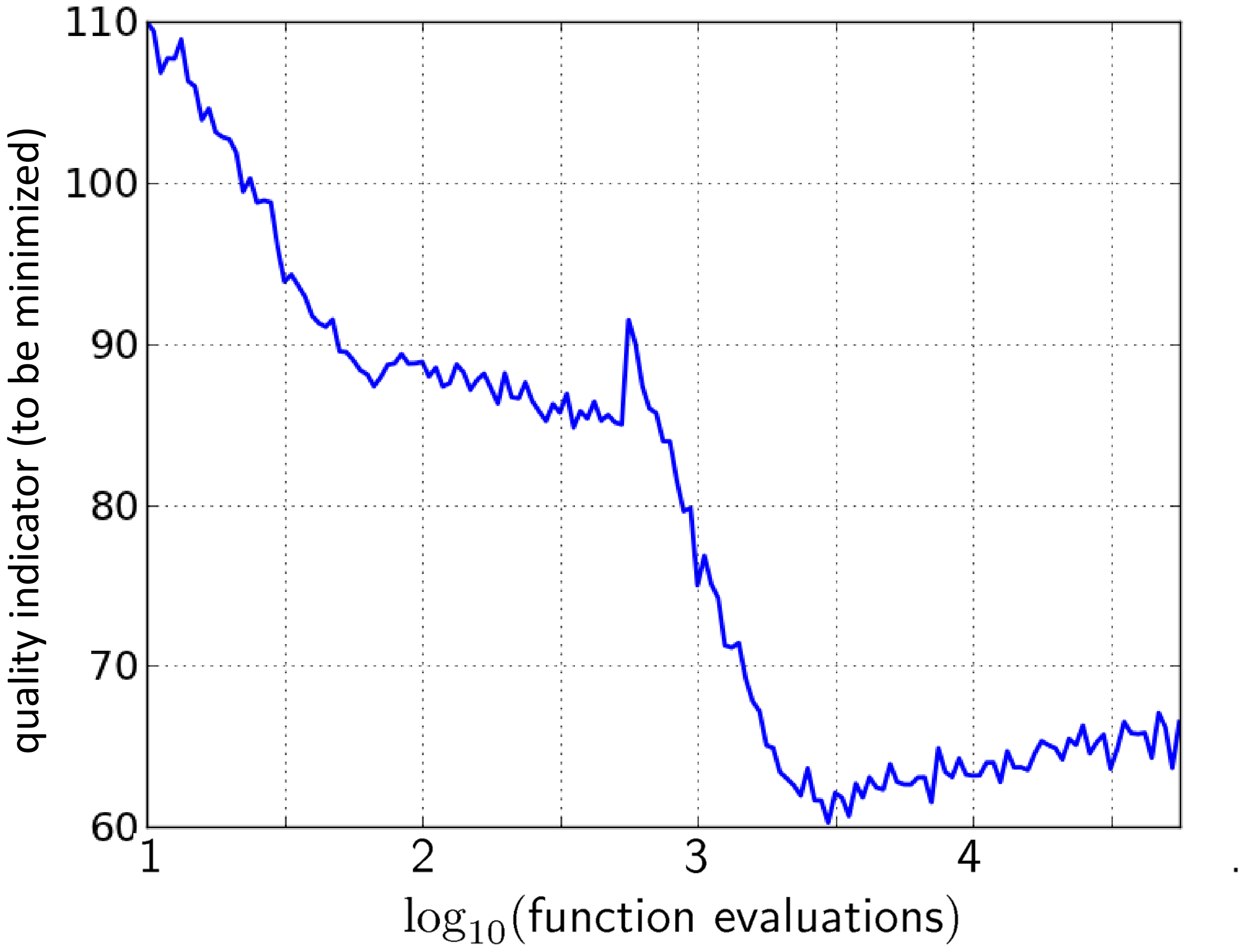


Advantage of fixed target view:
ratio scale (interpretation of #fevals easier than for f-values)

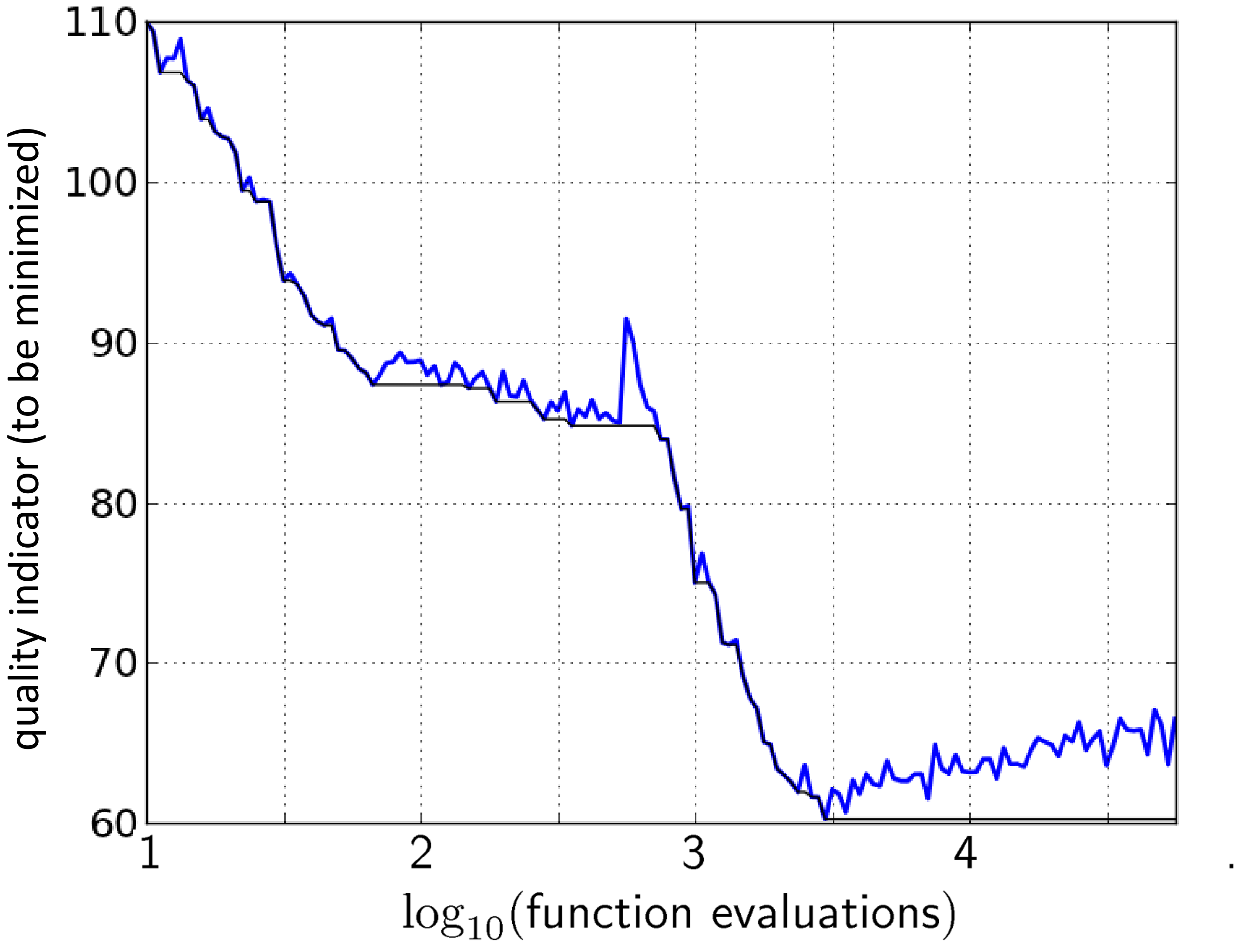
ECDF:

Empirical Cumulative Distribution Function of the Runtime

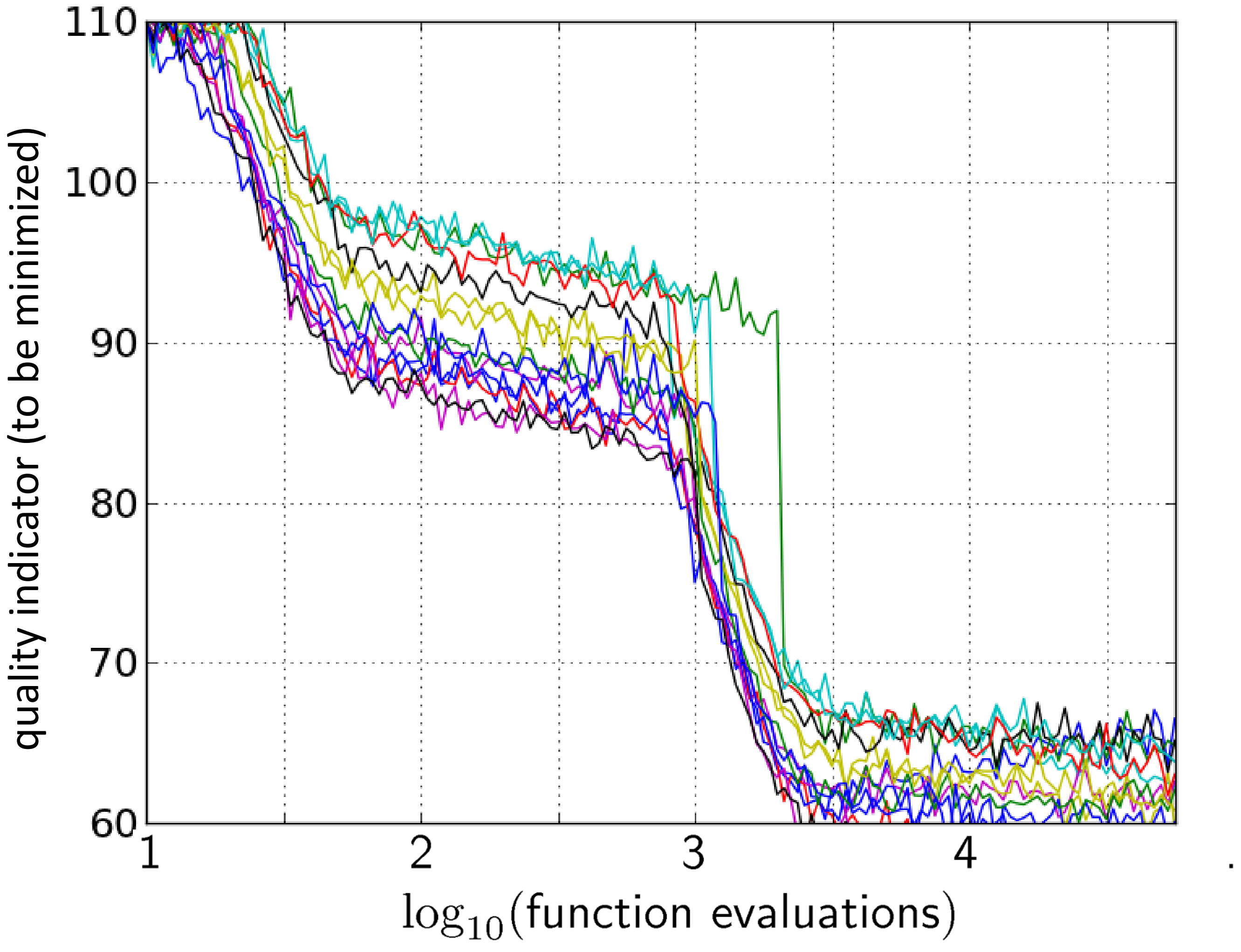
A Convergence Graph



First Hitting Time is Monotonous

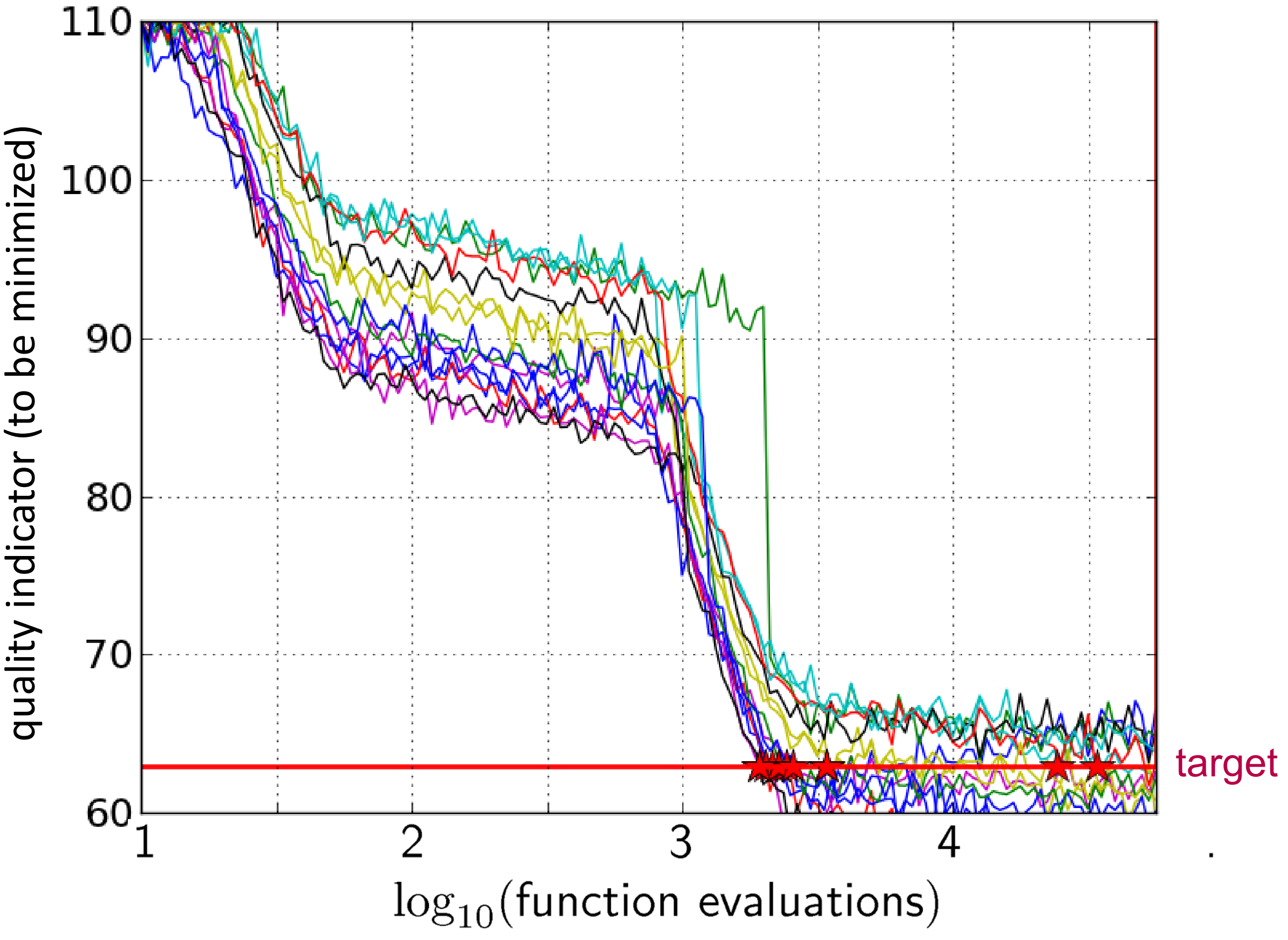


15 Runs

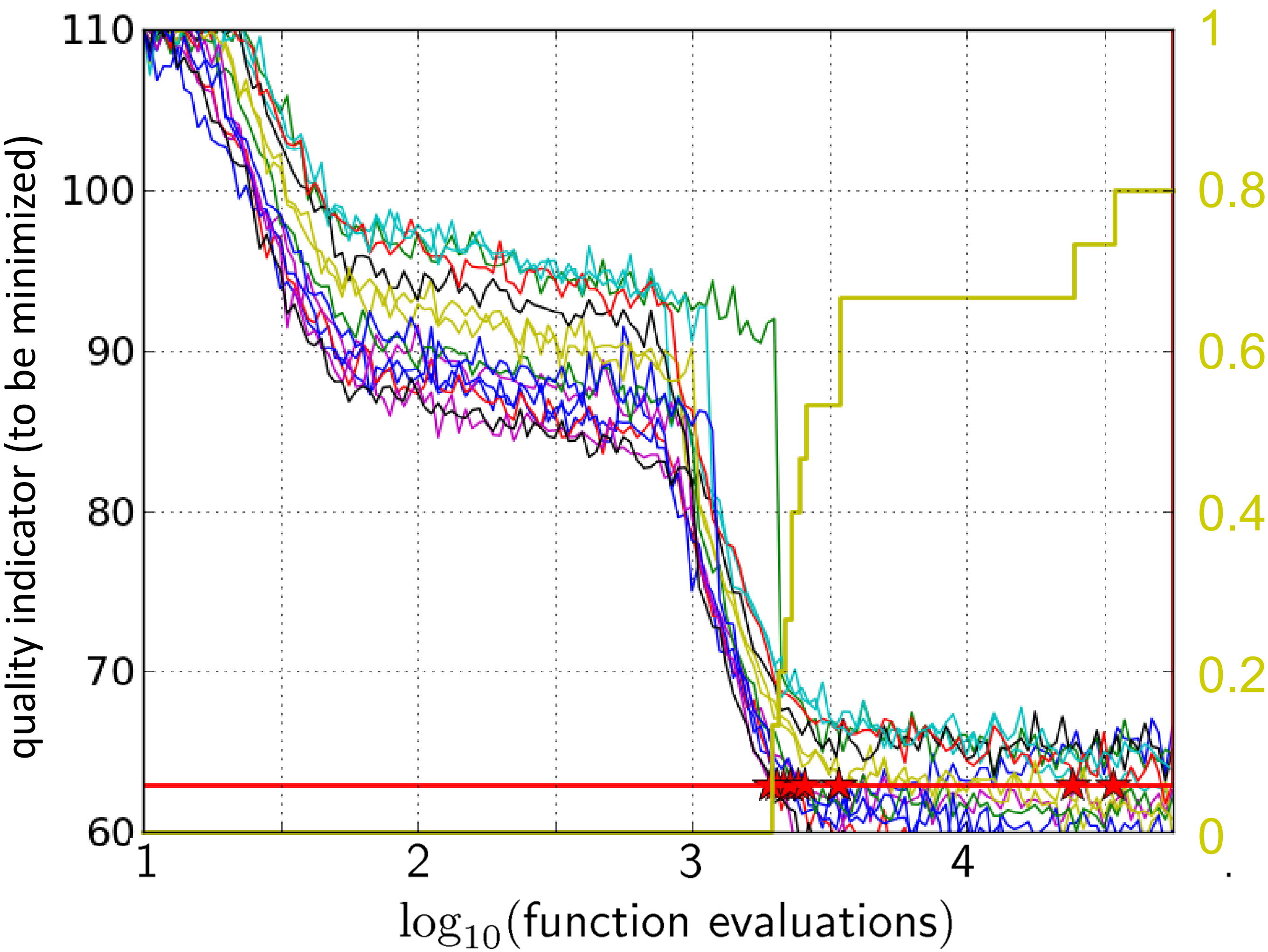


110
100
90
80
70
60
quality indicator (to be minimized)
1
2
3
4
log10(function evaluations)

15 Runs ≤ 15 Runtime Data Points



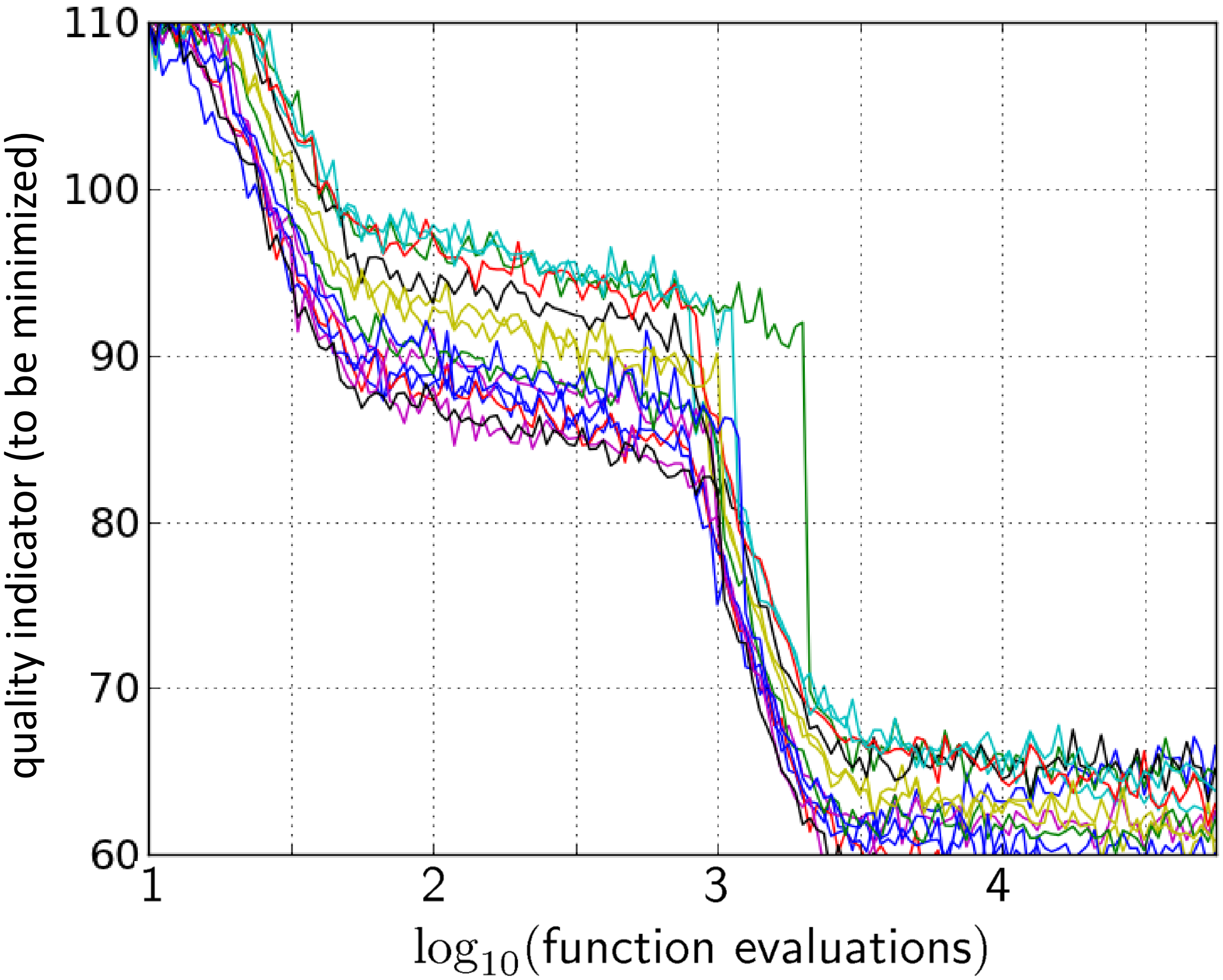
Empirical Cumulative Distribution



the ECDF of run lengths to reach the target

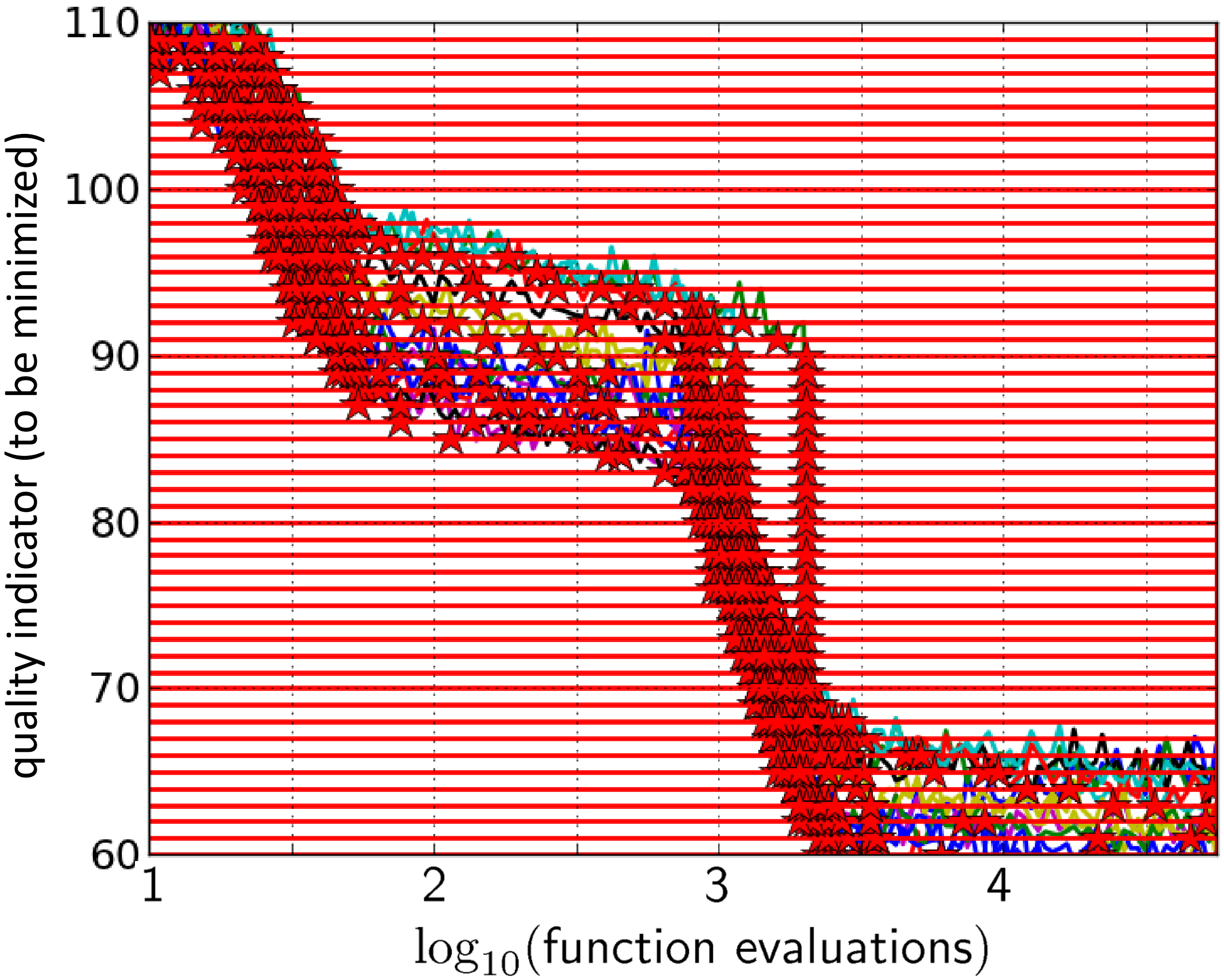
- has for each data point a vertical step of constant size
- displays for each x-value (budget) the count of observations to the left (first hitting times)

Aggregation



.

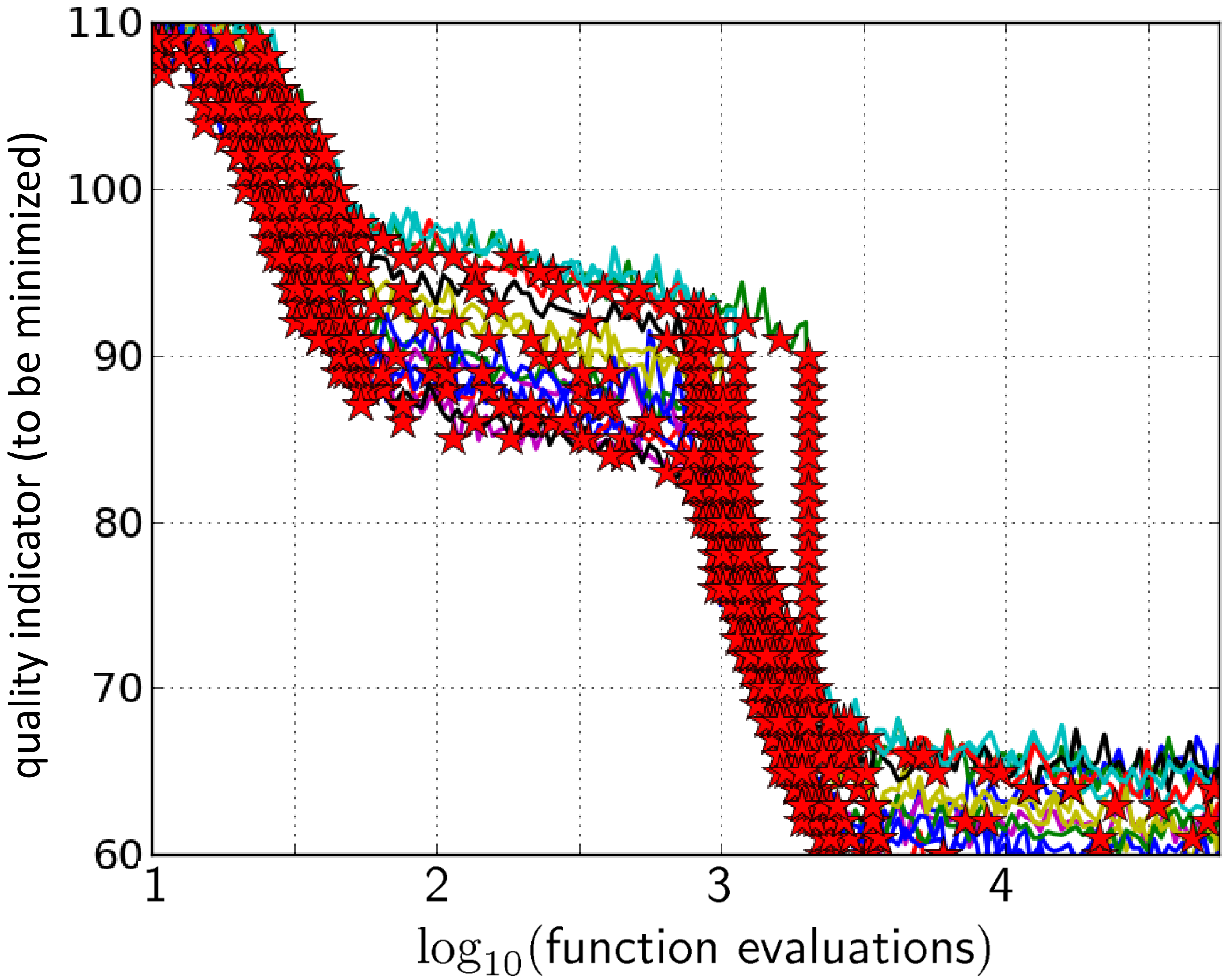
Aggregation



15 runs

50 targets

Aggregation



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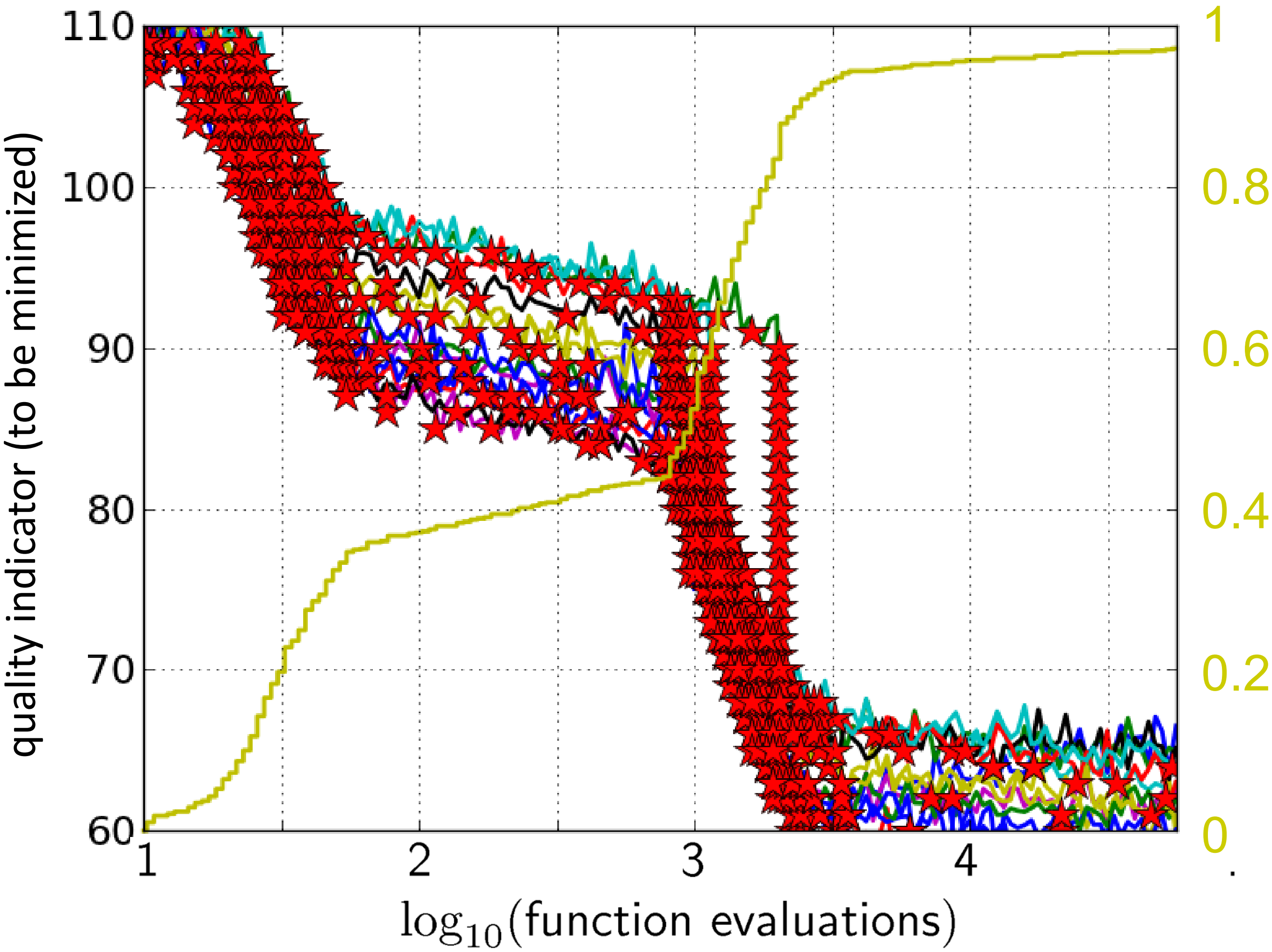
Aggregation

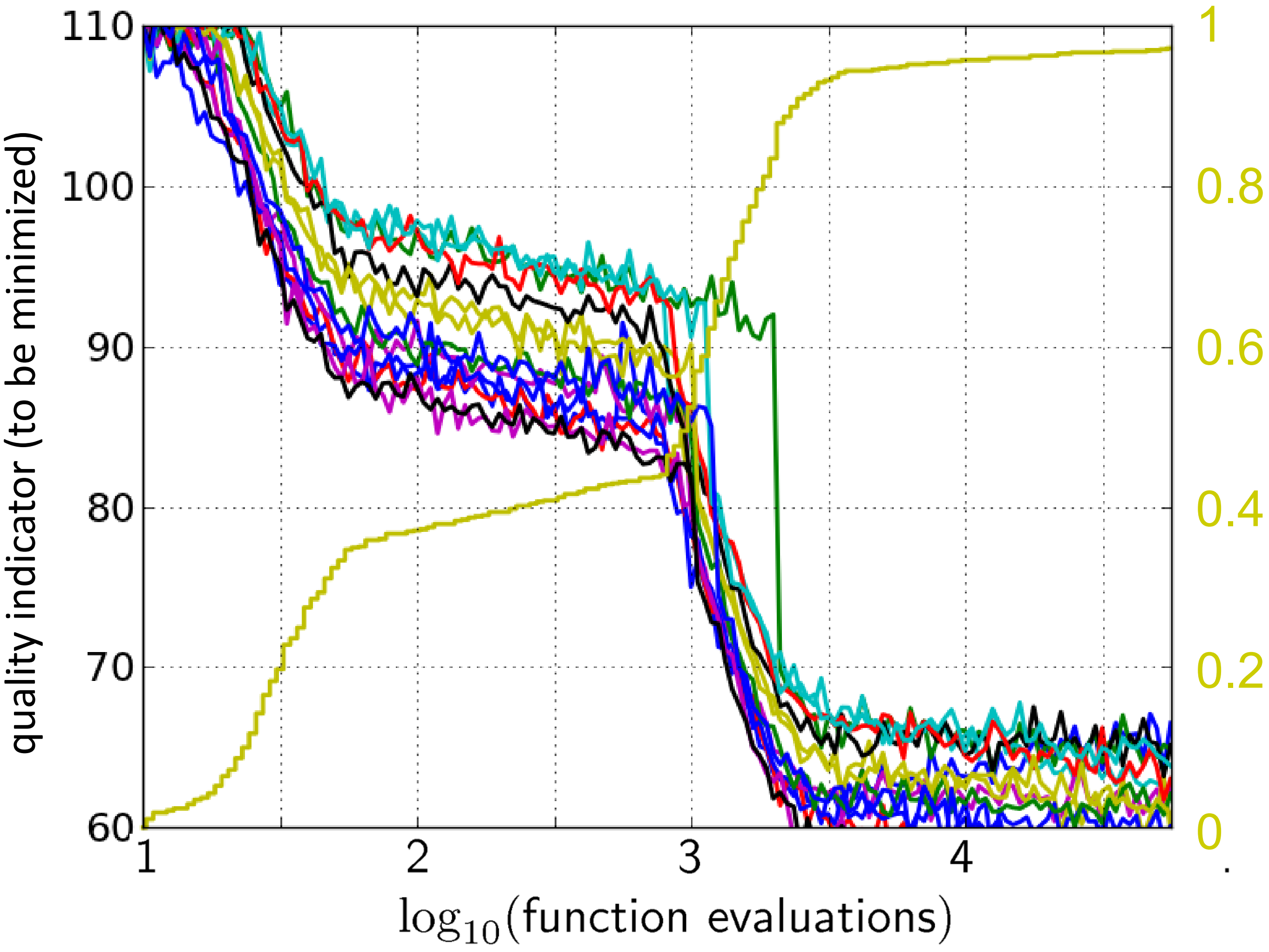


15 runs

50 targets

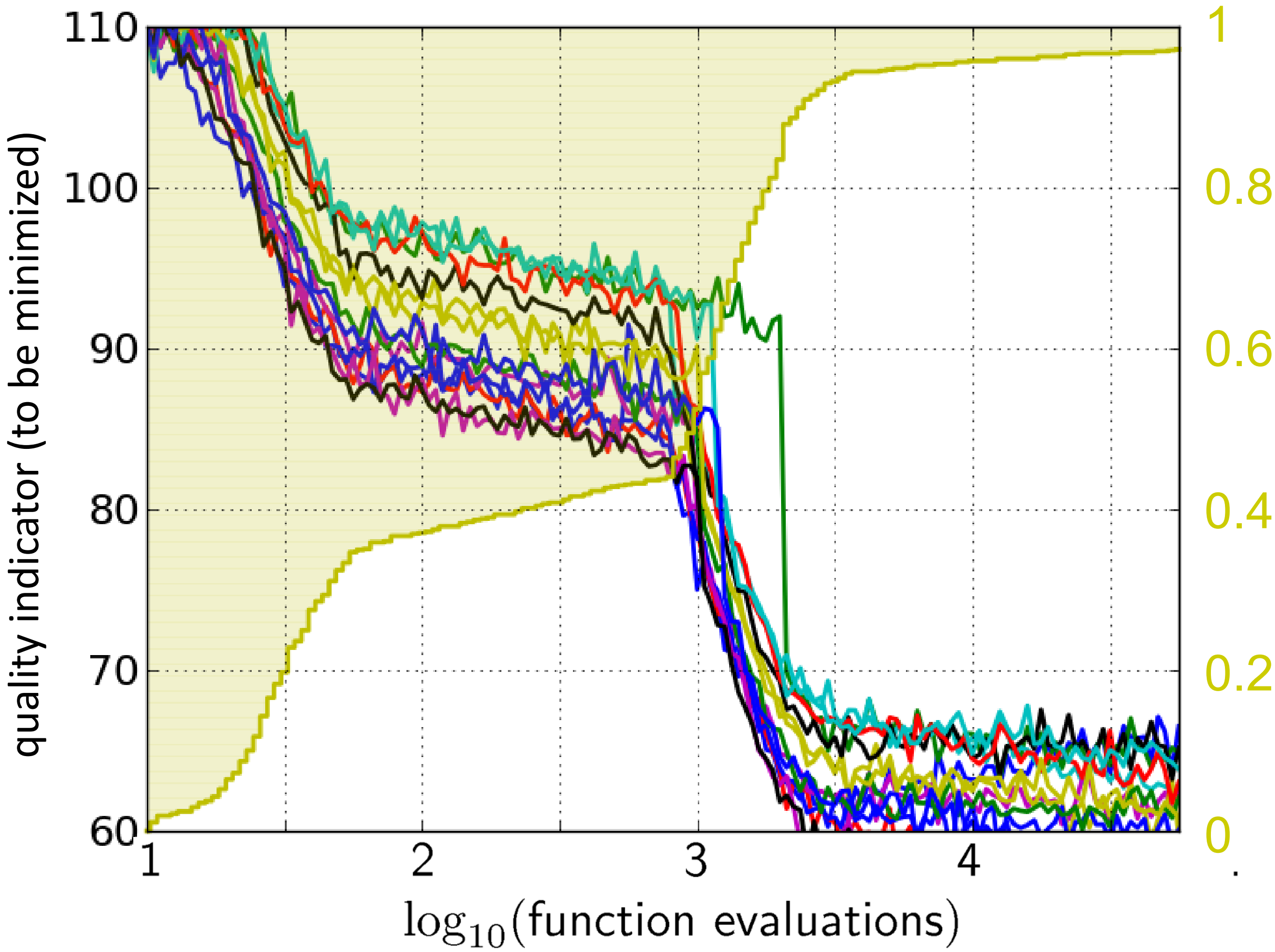
ECDF with
750 steps

Aggregation



50 targets from
15 runs
integrated in a
single graph

Interpretation



50 targets from 15 runs integrated in a single graph

area over the ECDF curve
=
average log runtime
(or geometric avg. runtime) over all targets (difficult and easy) and all runs

Worth to Note

ECDF graphs

- should never aggregate over dimension

dimension is input parameter to algorithm

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  - introduced by Moré and Wild [Moré and Wild 2009]
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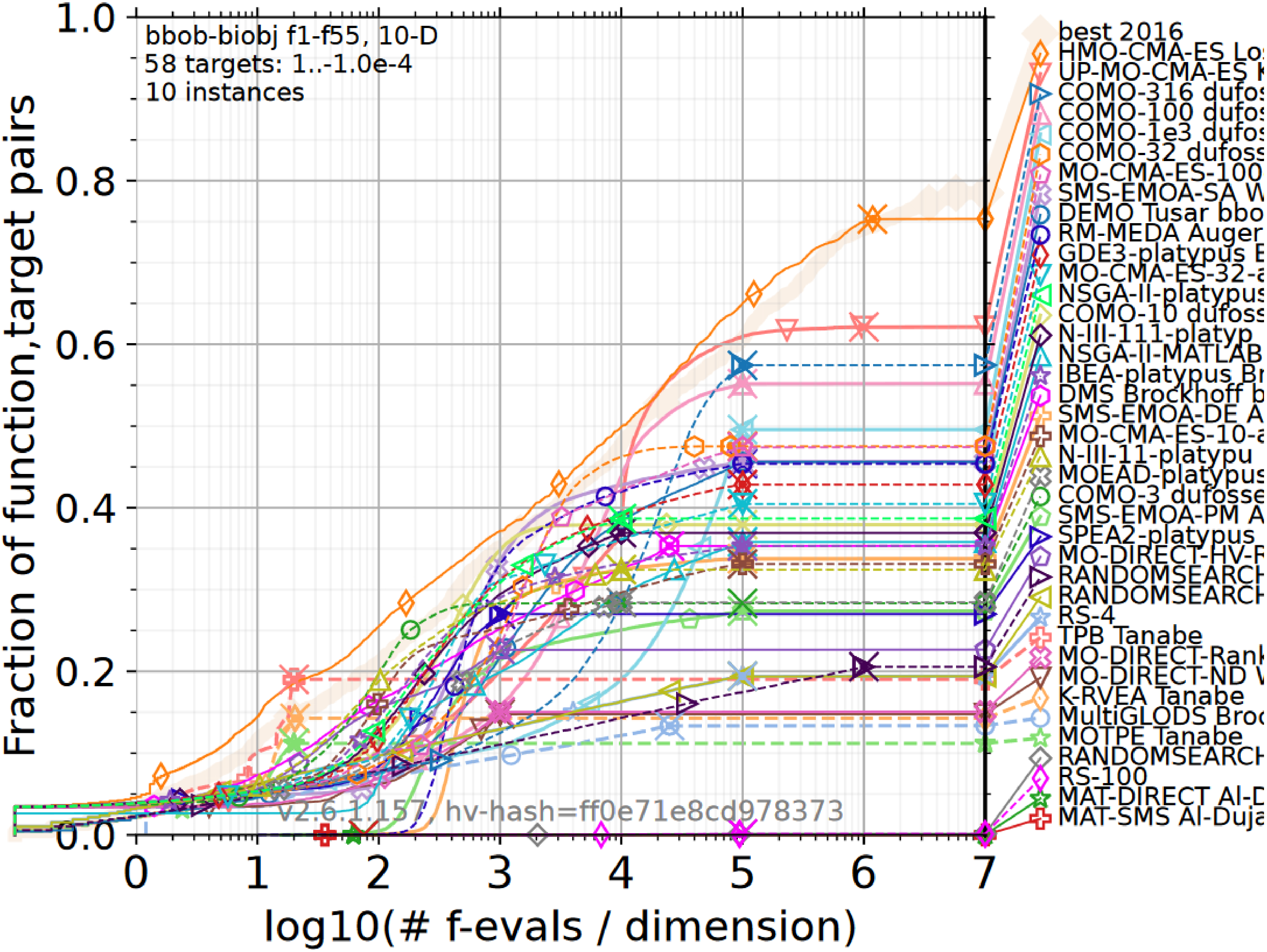
- but often over targets and functions
- can show data of more than 1 algorithm at a time
- are an extension of data profiles
  - introduced by Moré and Wild [Moré and Wild 2009]
  - but for multiple and absolute targets
- are COCO's main performance visualization tool

  [Hansen et al. 2021] - `https://github.com/numbbo/coco`

Example ECDF (later more)



Mostly Overlooked: Scaling with Dimension

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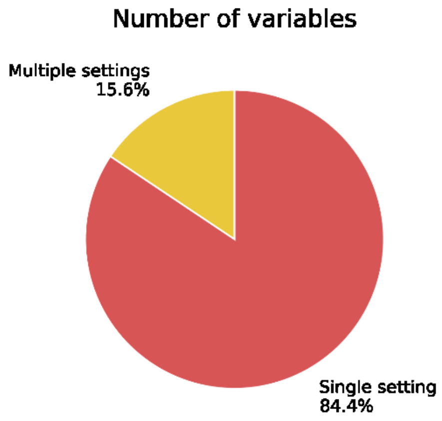

~10 papers from EMO'21 and PPSN/GECCO/CEC'20 change dimension

but 50+ papers have a "fixed" dimension

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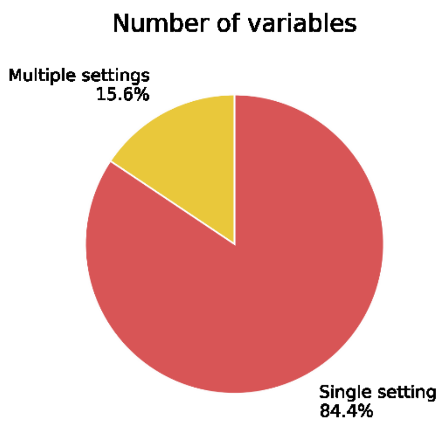

~10 papers from EMO'21 and PPSN/GECCO/CEC'20 change dimension

but 50+ papers have a "fixed" dimension

  - but in practice search space scalability almost more important

number of objectives often fixed

A Few General Recommendations

- always display everything you have
- look at single runs
- do each experiment at least twice

(= look at the *variance* of your results)

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A Few General Recommendations

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- look at single runs
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  (= look at the *variance* of your results)
- as quality indicators, use hypervolume, R2, or epsilon indicator

  or any indicator which is at least monotone
- see also the tutorial slides by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

http://www.cmap.polytechnique.fr/~nikolaus.hansen/gecco2018-experimentation-guide-slides.pdf

Recommended Experimental Setup (w/ or w/o COCO)

❶ **Benchmarking Experiment**

❷ **Choosing Algorithms for Comparison**

see `https://numbbo.github.io/data-archive/`

❸ **Postprocessing**

```
python -m cocopp resultfolder/ ALG2 ALG3
```

❹ **Displaying and Discussing Summary Results**

❺ **Investigating and Discussing Complementary Results**

❻ **Processed Data Sharing**

provide html output somewhere

❼ **Raw Data Sharing**

easy with COCO archive module & through issue tracker

Overview

❶ Performance Assessment

❷ Test Problems and Their Visualizations

❸ Recommendations from Numerical Results

Introduction

Test Problems (1)

Artificial problems (continuous and unconstrained)

- **v0.1:** Individual problems
- **v0.2:** MOP suite (unscalable problems)
- **v0.5:** ZDT suite (scalable number of variables)
- **v1.0:** DTLZ suite (scalable number of variables and objectives)
- **v1.2:** WFG suite
- **v1.3:** Other suites with a bottom-up construction
- **v1.5:** Suites of distance-based problems
- **v2.0:** The bbob-biobj(-ext) suite

Visualization of multiobjective landscapes

Low-dimensional search spaces

- Dominance ratio
- Local dominance
- Gradient path length

Any-dimensional search spaces

- Line cuts

Test Problems (2)

Artificial problems (other)

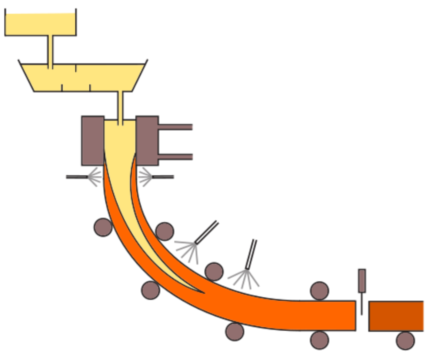
- Constrained problems
- Mixed-integer problems

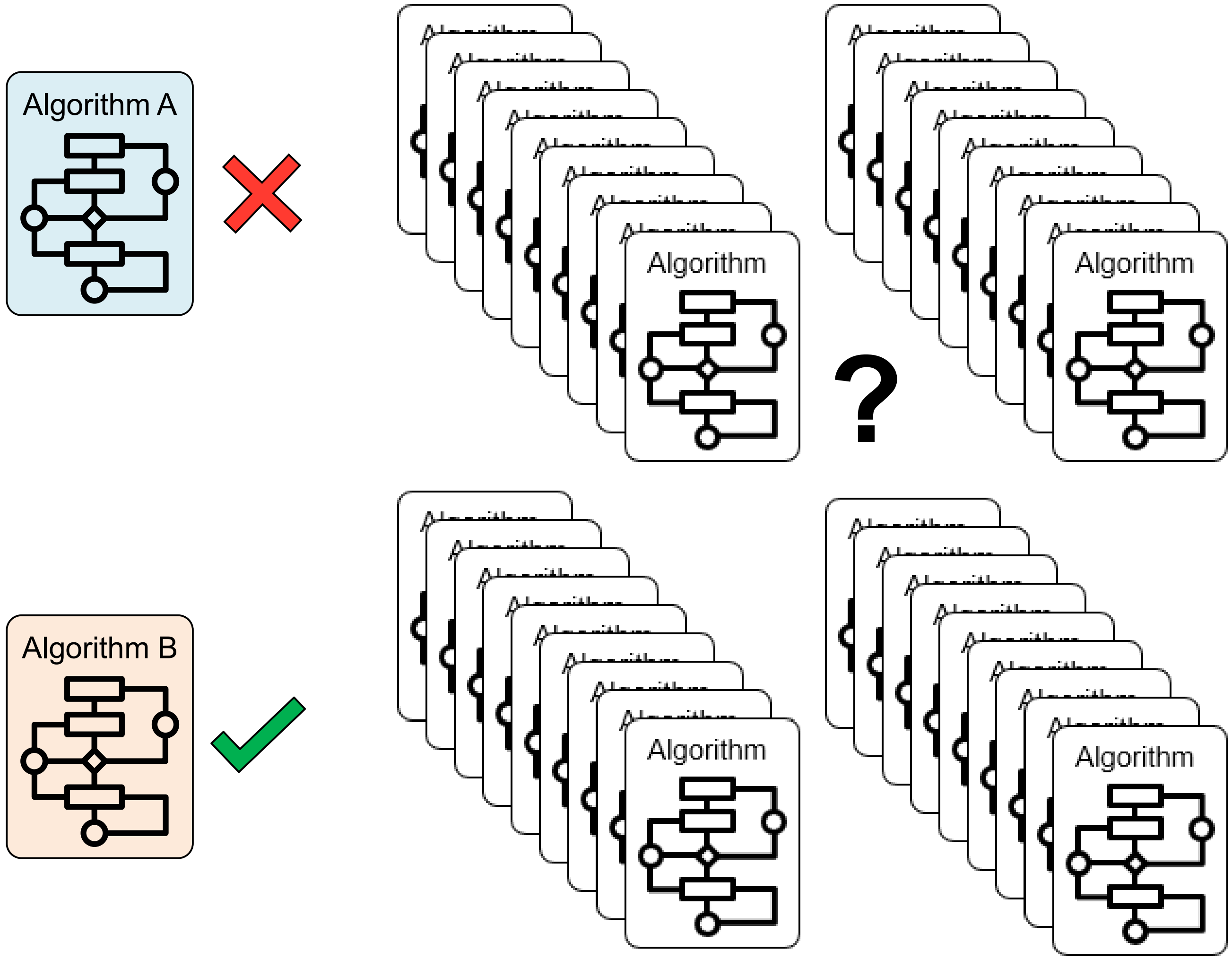
Real-world problems

- **v0.1:** Individual problems
- **v0.2:** Suites of unscalable problems
- **v0.5:** Suites of scalable problems (in the number of variables)

Conclusions

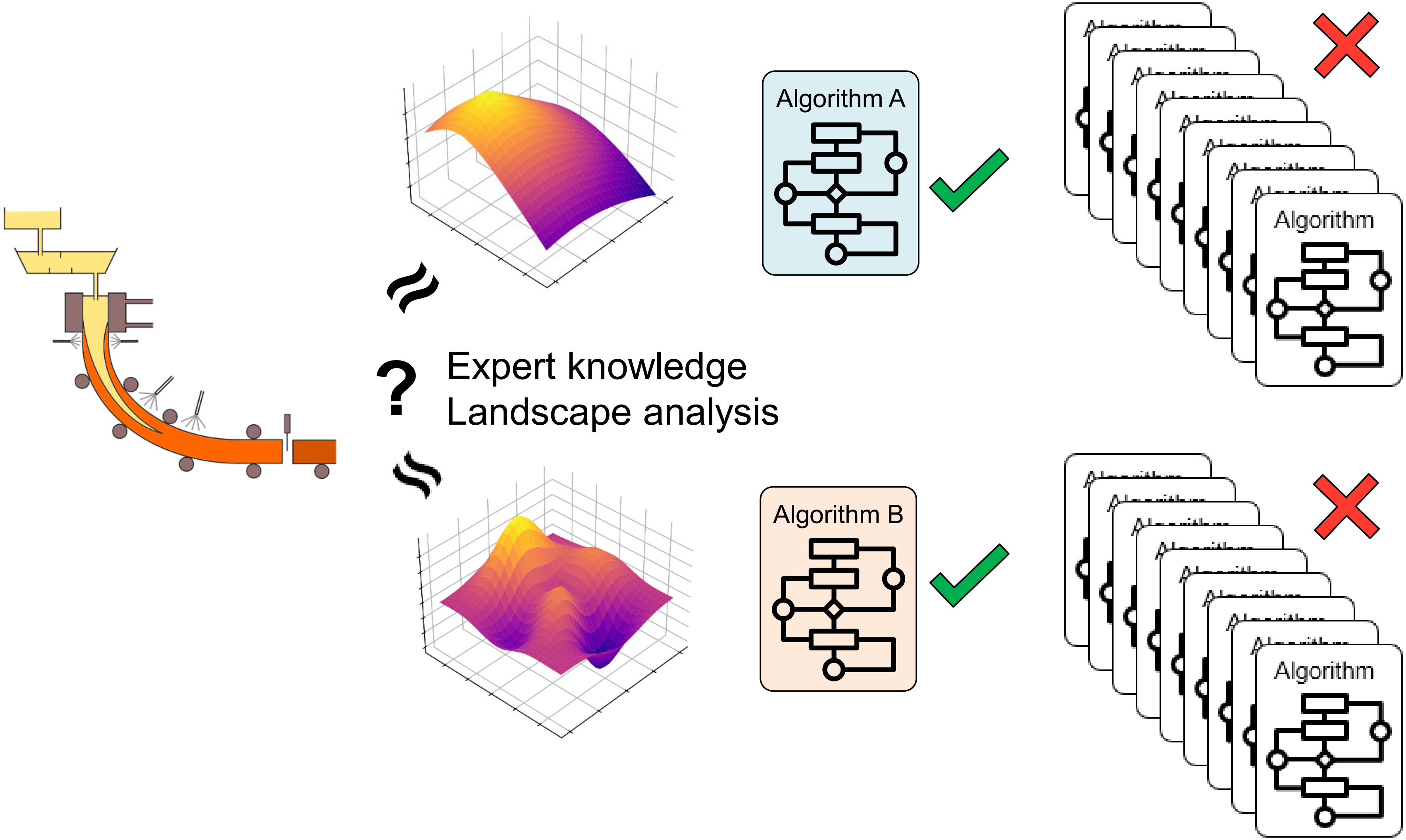
Why use test problems?





Introduction

Why use test problems?



Desirable characteristics of a benchmark problem set

[Bartz-Beielstein et al. 2020]

1. Diverse
2. Representative
3. Scalable and tunable
4. Known optima / best performance
5. [Continually updated]

Recommendations for multiobjective test suites

Adapted from [Huband et al. 2006]

1. A few “easy” (unimodal) test problems
2. The majority of problems should be hard (multimodal, nonseparable and both multimodal and nonseparable)
3. Diverse Pareto front geometries (including degenerate fronts, disconnected fronts) and disconnected Pareto sets

Additional recommendations for multiobjective test problems

Adapted from [Huband et al. 2006]

1. No extremal variables
2. No medial variables
3. Dissimilar variable domains
4. Dissimilar objective ranges

Problem Design Approaches

[Deb et al. 2005]

1. Multiple single-objective functions approach
2. Bottom-up approach
    1. Choose a Pareto front
    2. Build the objective space
    3. Construct the search space

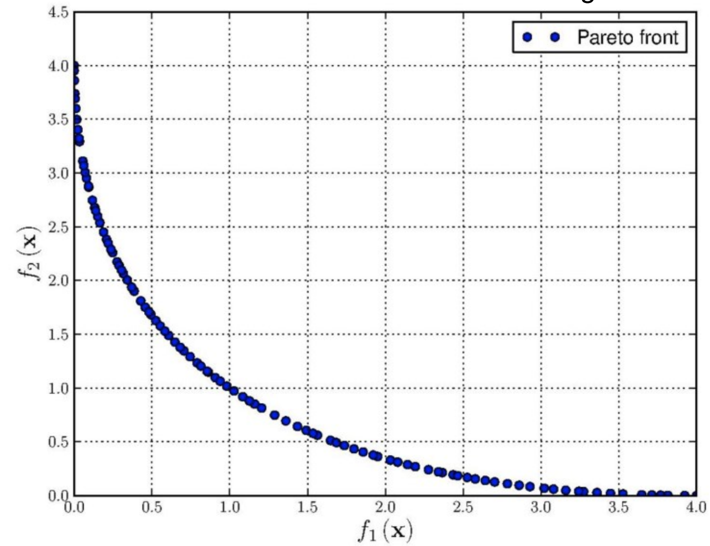
Artificial problems (continuous and unconstrained)

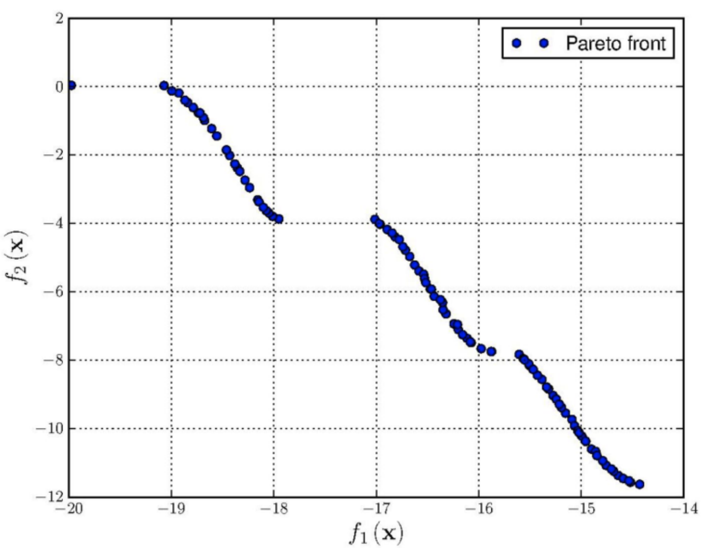
v0.1

Individual Problems

$$\text{Minimize} = \begin{cases} f_1(x) = x^2 \\ f_2(x) = (x-2)^2 \end{cases}$$

[Schaffer 1985]





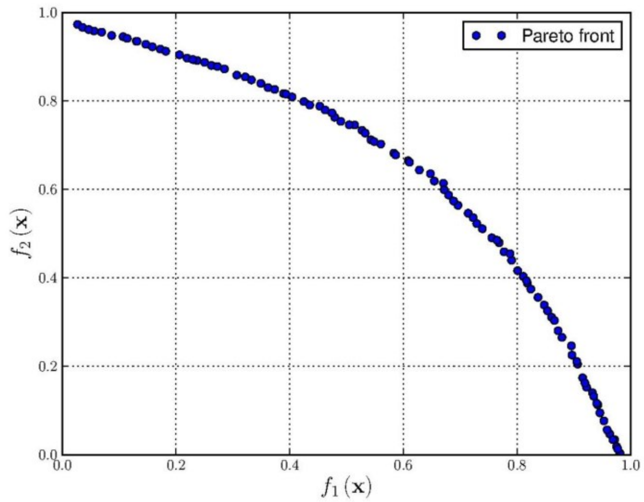
$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = \sum_{i=1}^{2} \left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right] \\ f_2(\boldsymbol{x}) = \sum_{i=1}^{3} \left[|x_i|^{0.8} + 5\sin\left(x_i^3\right)\right] \end{cases}$$

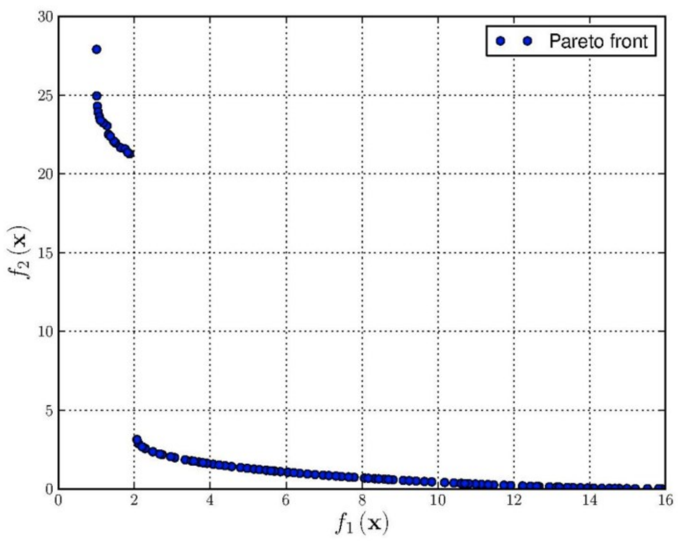
[Kursawe 1991]

Individual Problems

$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$$

[Fonseca and Fleming 1995]





$$\text{Minimize} = \begin{cases} f_1(x,y) = \left[1 + (A_1 - B_1(x,y))^2 + (A_2 - B_2(x,y))^2\right] \\ f_2(x,y) = (x+3)^2 + (y+1)^2 \end{cases}$$

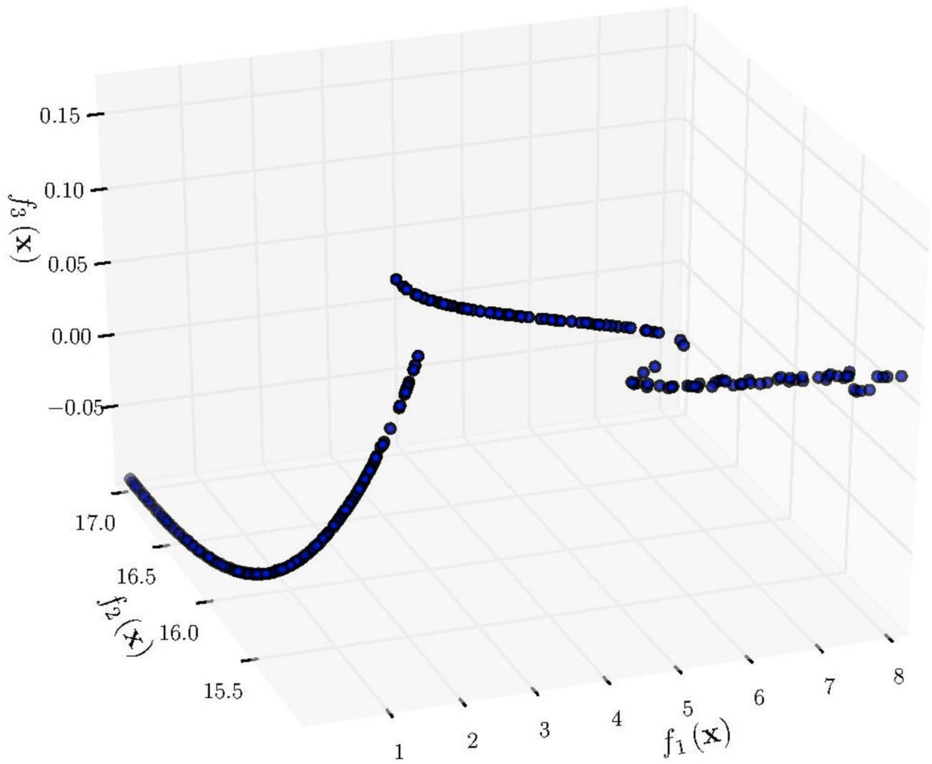
$$\text{where} = \begin{cases} A_1 = 0.5\sin(1) - 2\cos(1) + \sin(2) - 1.5\cos(2) \\ A_2 = 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2) \\ B_1(x,y) = 0.5\sin(x) - 2\cos(x) + \sin(y) - 1.5\cos(y) \\ B_2(x,y) = 1.5\sin(x) - \cos(x) + 2\sin(y) - 0.5\cos(y) \end{cases}$$

[Poloni et al. 1996]

Individual Problems

$$\text{Minimize} = \begin{cases} f_1(x,y) = 0.5\left(x^2+y^2\right) + \sin\left(x^2+y^2\right) \\ f_2(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\ f_3(x,y) = \frac{1}{x^2+y^2+1} - 1.1\exp\left(-\left(x^2+y^2\right)\right) \end{cases}$$

[Viennet et al. 1996]



v0.2

MOP Suite

MOP = Multi-Objective Problem

[Van Veldhuizen 1999]

Properties

- A collection of 7 test problems from the literature
- Some problems are both nonseparable and multimodal
- A collection of various Pareto front geometries

Issues

- Most problems have 2 or 3 variables
- Not scalable in the number of objectives
- In many problems the optima lie on the boundary or in the middle of the search space
- The Pareto set is hard to compute for some problems

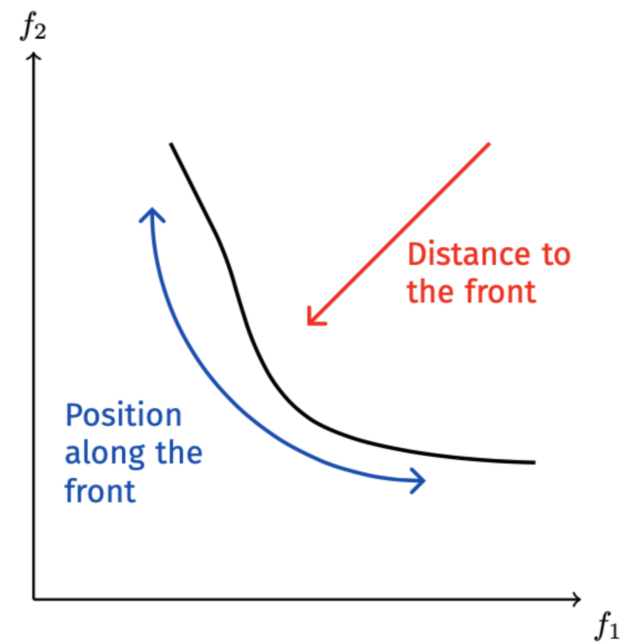
v0.5

ZDT Suite

ZDT = Zitzler, Deb, Thiele

[Zitzler et al. 2000]

Construction with the bottom-up approach (following Deb's toolkit [Deb 1999])



Given $\mathbf{x} = \{x_1, \ldots, x_n\}$

Distribution f.

Minimise $f_1(\mathbf{y})$

Distance f. Front shape

$$f_2(\mathbf{y}, \mathbf{z}) = g(\mathbf{z})h(f_1(\mathbf{y}), g(\mathbf{z}))$$

where $\mathbf{y} = \{x_1, \ldots, x_j\}$ Position variable(s) (j = 1 for ZDT)

$\mathbf{z} = \{x_{j+1}, \ldots, x_n\}$ Distance variables

The separation of variables was done to simplify problem construction

ZDT Suite

Properties

- Scalable in the number of (distance) variables
- Some problems are multimodal
- Convex, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known

Issues

- Not scalable in the number of objectives (2 objectives)
- 4 problems have optima on the boundary of the search space
- 1 problem has optima in the middle of the search space
- All problems are separable (the first objective depends only on the first variable)

v1.0

DTLZ Suite

DTLZ = Deb, Thiele, Laumanns, Zitzler

[Deb et al. 2005]

Improvement over ZDT

- Scalable number of objectives
- Optima do not lie on the boundary of the search space

Remaining issues

- Most problems have optima in the middle of the search space
- Problems still separable in practice (optimizing one variable at a time will yield at least one global optimum)

v1.2

WFG Suite

WFG = Walking Fish Group

[Huband et al. 2006]

Improvement over DTLZ

- Optima do not lie in the middle of the search space
- Some nonseparable, multimodal, deceptive and biased problems
- Convex, linear, concave, mixed, disconnected and degenerate Pareto fronts

Remaining issues

- The Pareto set is linear for 8 of the 9 problems
- Still rely on distance and position variables

v1.3

Other Suites and Problems

Problems constructed with the bottom-up approach

[Zapotecas et al. 2019]

- L-ZDT and L-DTLZ problems with linkages [Deb et al. 2006]
- IHR test suite of 5 rotated ZDT problems [Igel et al. 2007]
- LZ test suite of 9 problems with complicated Pareto sets [Li and Zhang 2009]
- SZDT test suite of 7 scalable problems with complicated Pareto sets [Saxena et al. 2011]
- Convex DTLZ problem [Deb and Jain 2014]
- Inverted DTLZ problem [Jain and Deb 2014]
- MNI test suite of 2 test problems with diverse shapes of the Pareto front [Masuda et al. 2016]
- LSMOP test suite of 9 test problems for large-scale optimization with variable dependencies [Cheng et al. 2017b]
- Minus-DTLZ and Minus-WFG test suites [Ishibuchi et al. 2017]
- MMF test problems with diverse landscapes [Yue et al. 2019]

CEC Competition Suites

Information about all CEC competitions:
https://www3.ntu.edu.sg/home/EPNSugan/index_files/cec-benchmarking.htm

- 13 test problems for CEC 2007 [Huang et al. 2007]
  - OKA [Okabe et al. 2004], SYM-PART [Rudolph et al. 2007]
  - 4 shifted ZDT, 1 rotated ZDT
  - 2 shifted DTLZ, 1 rotated DTLZ
  - 3 WFG

- 13 test problems for CEC 2009 (UF suite) [Zhang et al. 2009]
  - 10 with complicated Pareto sets (4 from the LZ suite)
  - 2 extended rotated DTLZ
  - 1 WFG

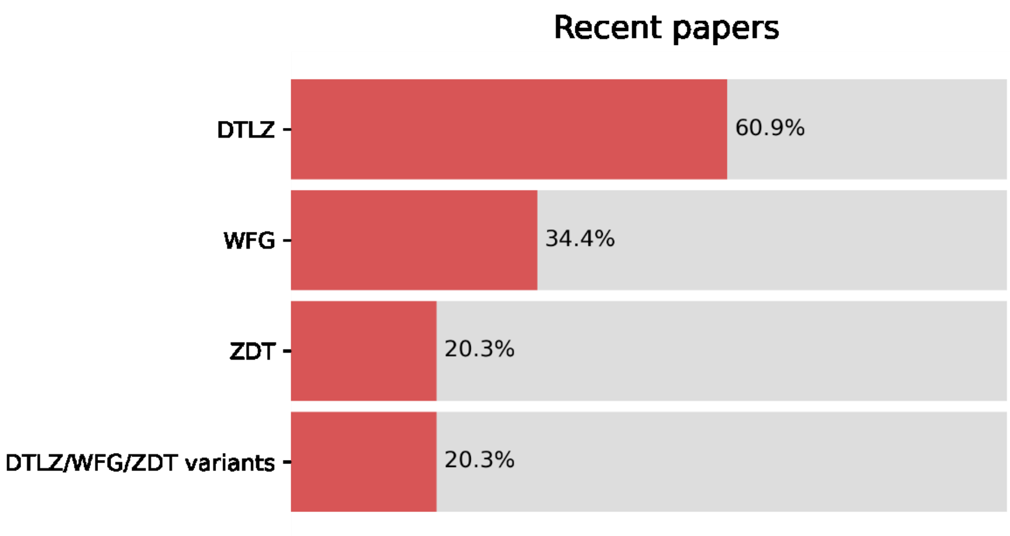
CEC Competition Suites

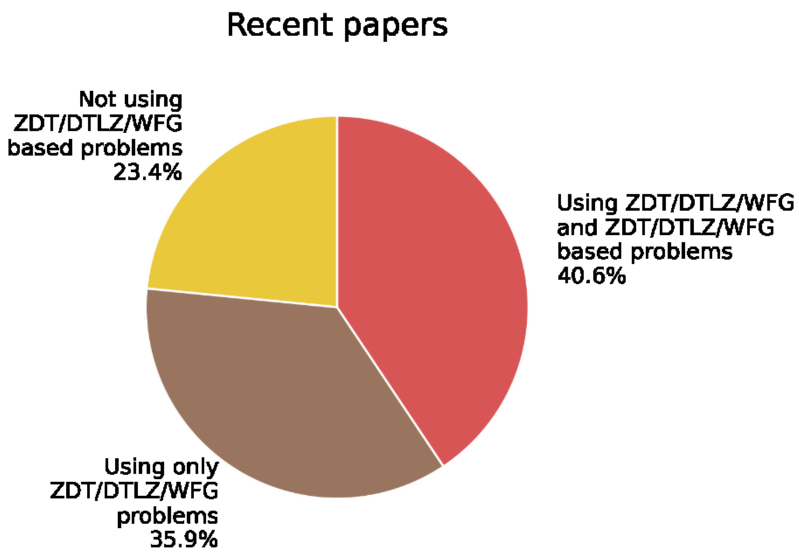
- 15 test problems for CEC 2017 (MaF suite) [Cheng et al. 2017a]
  - 7 modified DTLZ problems
  - 2 distance minimization problems
  - 3 WFG problems
  - 1 SZDT problem
  - 2 LSMOP problem
- 22 test problems for CEC 2019 [Liang et al. 2019]
  - 2 SYM-PART
  - Omni-test [Deb and Tiwari 2008]
  - 19 MMF problems
- 24 test problems for CEC 2020 [Liang et al. 2020]
  - 24 MMF problems

Survey of Recent Papers

64 papers on unconstrained continuous multiobjective optimization from recent conferences (without application papers)

- CEC 2020
- GECCO 2020
- PPSN 2020
- EMO 2021





v1.5

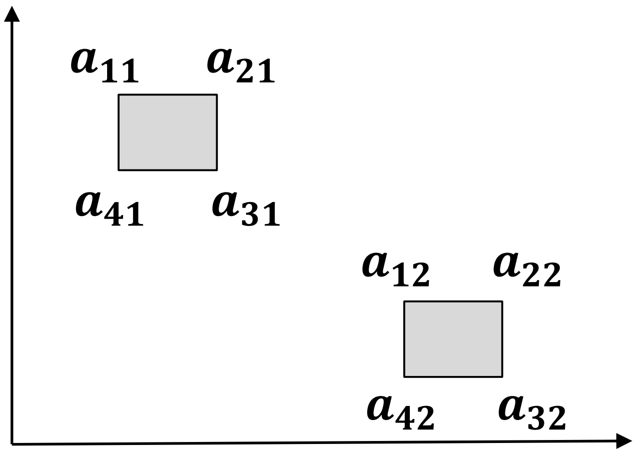
Distance-Based Problems

General idea

[Ishibuchi et al. 2010] based on earlier work [Köppen et al. 2005, Rudolph et al. 2007]

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ \ldots,\ f_k(\mathbf{x}))$$

$$f_i(\mathbf{x}) = \min\{\text{dis}(\mathbf{x}, \mathbf{a}_{i1}), \text{dis}(\mathbf{x}, \mathbf{a}_{i2}), \ldots, \text{dis}(\mathbf{x}, \mathbf{a}_{im})\}$$



Properties

- 2-D test problems that are inherently visualizable
- Pareto set easy to characterize
- Scalable in the number of objectives
- Useful for visualizing the distribution of solutions

Issues

- Simple objective functions

Extensions

- High-dimensional search spaces [Masuda et al. 2014]
- Distance to lines (instead of points) [Li et al. 2014, 2018]
- Dominance resistance regions [Fieldsend 2016]
- Local Pareto fronts [Liu et al. 2018]
- Problem generator for scalable problems with various properties (local fronts, disconnected Pareto sets and fronts, dominance resistance regions, uneven ranges of objective values, varying density of solutions) [Fieldsend et al. 2019]

v2.0

Motivation

[Brockhoff et al. 2022]

- Real-world problems are not constructed using the bottom-up approach
- Go back to basics – use single-objective functions for each objective
- Idea not new [Schaffer 1985, Igel et al. 2007, Emmerich and Deutz 2007, Kerschke et al. 2016]

bbob-biobj Suite

Construction

- Use the functions from the **bbob** suite [Finck et al. 2009]
  - Well-understood
  - Scalable in the number of variables and parametrized (instances)
  - 24 functions categorized in 5 groups based on their properties
    - Separable
    - Low or moderate conditioning
    - High conditioning and unimodal
    - Multimodal with global structure
    - Multimodal with weak global structure
- How to avoid an explosion in the number of problems?

bbob-biobj Suite

	Separable		Low or moderate conditioning		High conditioning and unimodal		Multimodal with global structure		Multimodal with weak global structure		
	f_1	f_2	f_6	f_8	f_{13}	f_{14}	f_{15}	f_{17}	f_{20}	f_{21}	
f_1	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	Separable
f_2		F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}	Separable
f_6			F_{20}	F_{21}	F_{22}	F_{23}	F_{24}	F_{25}	F_{26}	F_{27}	Low or moderate conditioning
f_8				F_{28}	F_{29}	F_{30}	F_{31}	F_{32}	F_{33}	F_{34}	Low or moderate conditioning
f_{13}					F_{35}	F_{36}	F_{37}	F_{38}	F_{39}	F_{40}	High conditioning and unimodal
f_{14}						F_{41}	F_{42}	F_{43}	F_{44}	F_{45}	High conditioning and unimodal
f_{15}							F_{46}	F_{47}	F_{48}	F_{49}	Multimodal with global structure
f_{17}								F_{50}	F_{51}	F_{52}	Multimodal with global structure
f_{20}									F_{53}	F_{54}	Multimodal with weak global structure
f_{21}										F_{55}	Multimodal with weak global structure

- f_1 Sphere
- f_2 Ellipsoid separable
- f_6 Attractive sector
- f_8 Rosenbrock original
- f_{13} Sharp ridge
- f_{14} Sum of different powers
- f_{15} Rastrigin
- f_{17} Schaffer F7 (condition 10)
- f_{20} Schwefel x*sin(x)
- f_{21} Gallagher 101 peaks

	Separable					Low or moderate conditioning				High conditioning and unimodal					Multimodal with global structure					Multimodal with weak global structure					
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}	
Separable	F_1	F_2	F_{56}	F_{57}	F_{58}	F_3		F_4					F_5	F_6	F_7		F_8			F_9	F_{10}				f_1
		F_{11}	F_{59}	F_{60}	F_{61}	F_{12}		F_{13}					F_{14}	F_{15}	F_{16}		F_{17}			F_{18}	F_{19}				f_2
				F_{62}	F_{63}																				f_3
					F_{64}																				f_4
																									f_5
Low or moderate conditioning						F_{20}	F_{65}	F_{21}	F_{66}				F_{22}	F_{23}	F_{24}		F_{25}			F_{26}	F_{27}				f_6
								F_{67}	F_{68}																f_7
								F_{28}	F_{69}				F_{29}	F_{30}	F_{31}		F_{32}			F_{33}	F_{34}				f_8
																									f_9
High conditioning and unimodal											F_{70}	F_{71}	F_{72}	F_{73}											f_{10}
												F_{74}	F_{75}	F_{76}											f_{11}
													F_{77}	F_{78}											f_{12}
													F_{35}	F_{36}	F_{37}		F_{38}			F_{39}	F_{40}				f_{13}
														F_{41}	F_{42}		F_{43}			F_{44}	F_{45}				f_{14}
Multimodal with global structure															F_{46}		F_{47}	F_{79}	F_{80}	F_{48}	F_{49}				f_{15}
																									f_{16}
																	F_{50}	F_{81}	F_{82}	F_{51}	F_{52}				f_{17}
																			F_{83}						f_{18}
																									f_{19}
Multimodal with weak global structure																				F_{53}	F_{54}	F_{84}	F_{85}	F_{86}	f_{20}
																					F_{55}	F_{87}	F_{88}	F_{89}	f_{21}
																							F_{90}	F_{91}	f_{22}
																								F_{92}	f_{23}
																									f_{24}

- f_1 Sphere
- f_2 Ellipsoid separable
- f_3 Rastrigin separable
- f_4 Skew Rastrigin-Bueche
- f_5 Linear slope

- f_6 Attractive sector
- f_7 Step-ellipsoid
- f_8 Rosenbrock original
- f_9 Rosenbrock rotated

- f_{10} Ellipsoid
- f_{11} Discus
- f_{12} Bent cigar
- f_{13} Sharp ridge
- f_{14} Sum of different powers

- f_{15} Rastrigin
- f_{16} Weierstrass
- f_{17} Schaffer F7 (condition 10)
- f_{18} Schaffer F7 (condition 1000)
- f_{19} Griewank-Rosenbrock F8F2

- f_{20} Schwefel x*sin(x)
- f_{21} Gallagher 101 peaks
- f_{22} Gallagher 21 peaks
- f_{23} Katsuuras
- f_{24} Lunacek bi-Rastrigin

bbob-biobj(-ext) Suites

Properties

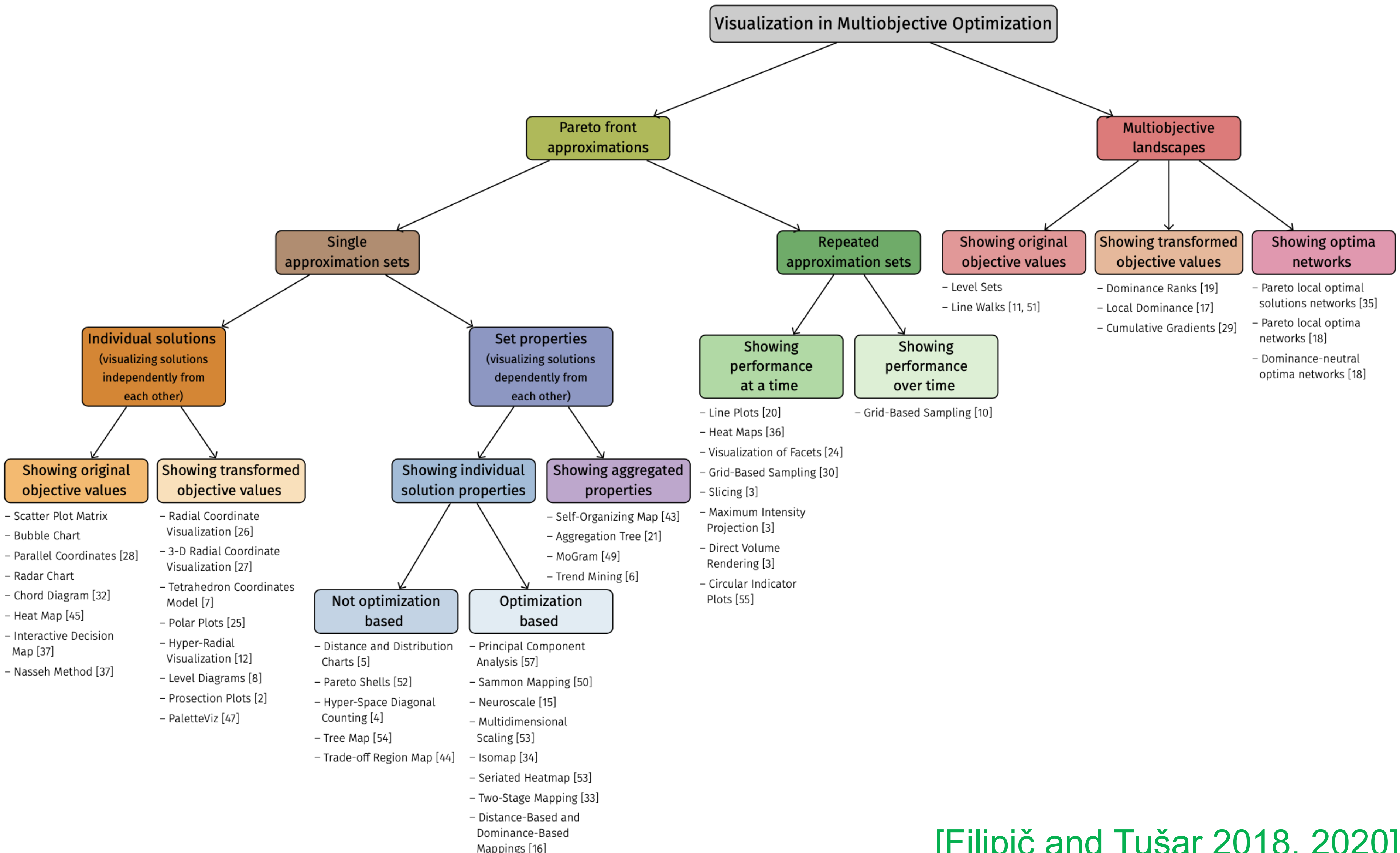
- Construction similar as in real-world problems
- Scalability in the number of variables
- Various problem properties (more diverse than existing multiobjective test suites)
- Included in the COCO benchmarking platform [Hansen et al. 2021]
- Problem instances can be quite diverse

Issues

- Only 2 objectives
- Unknown Pareto set and front, but known single-objective optima and available approximations of the Pareto fronts (and sets for lower-dimensional problems)

Visualization of multiobjective landscapes

Visualization in Multiobjective Optimization



[Filipič and Tušar 2018, 2020]

Visualization of Multiobjective Problem Landscapes

Low-dimensional search spaces

- Dominance ratio [Fonseca 1995]
- Gradient path length (inspired by gradient plots [Kerschke and Grimme 2017])
- Local dominance [Fieldsend et al. 2019]
- PLOT [Shaepermeier et al. 2020]

Any-dimensional search spaces

- Line cuts [Brockhoff et al. 2022, Volz et al. 2019]
- Optima network [Liefooghe et al. 2018, Fieldsend and Alyahya 2019]

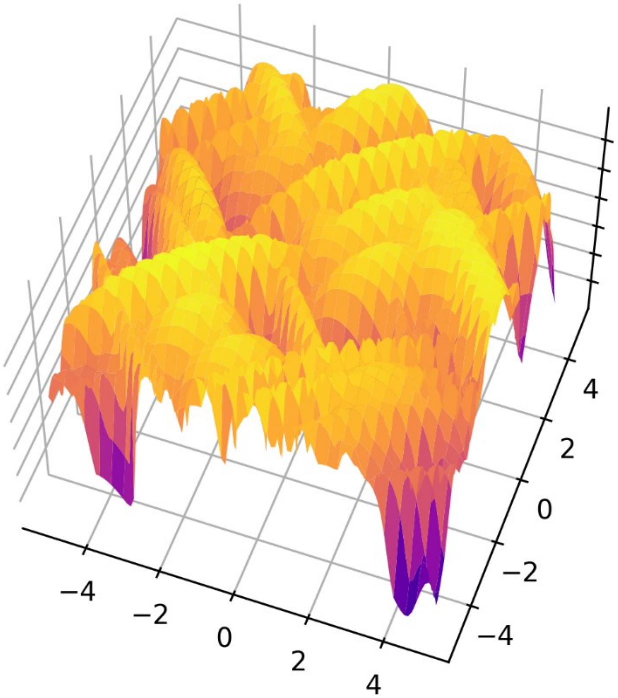
Various visualizations of bbob-biobj-ext problems:
https://numbbo.github.io/bbob-biobj/

Visualizations of bbob-biobj and other multi-objective suites using PLOT:
https://schaepermeier.shinyapps.io/moPLOT/

Problems for demonstration

- Double sphere problem bbob-biobj $F_1 = (f_1, f_1)$, instance 1
- Sphere-Gallagher problem bbob-biobj $F_{10} = (f_1, f_{21})$, instance 1
- Double Gallagher problem bbob-biobj $F_{55} = (f_{21}, f_{21})$, instance 1

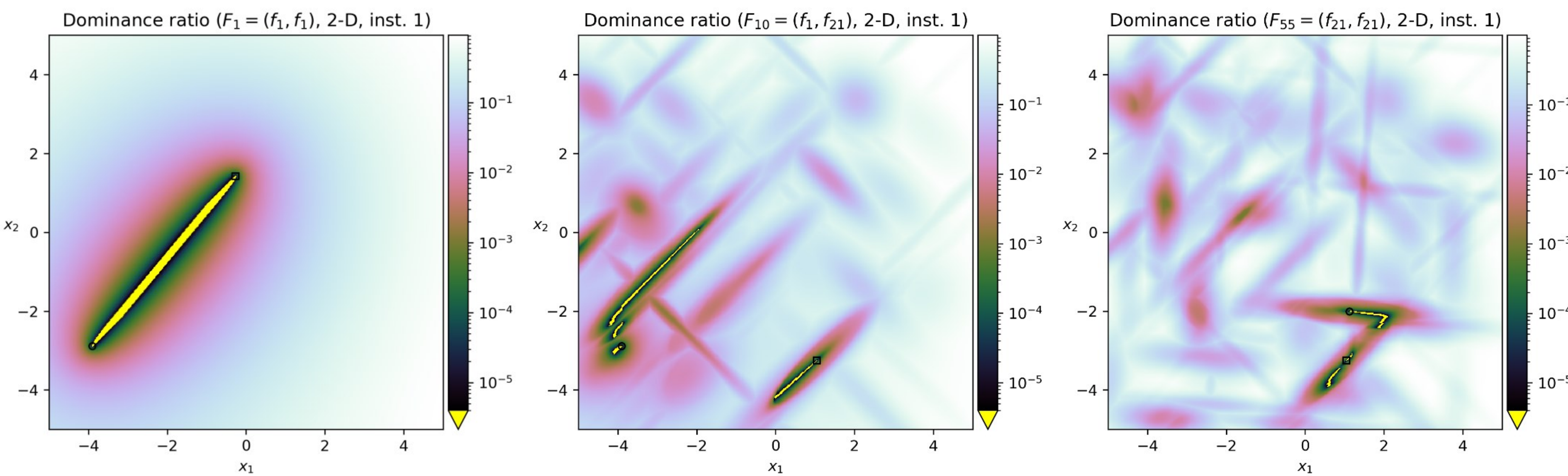
Gallagher = Gallagher's Gaussian
101-me Peaks Function



Dominance Ratio

[Fonseca 1995]

- Discretized search space (501 x 501 grid)
- Dominance ratio = the ratio of grid points that dominate the current point
- All nondominated points have a ratio of zero
- Visualize dominance ratios in logarithmic scale



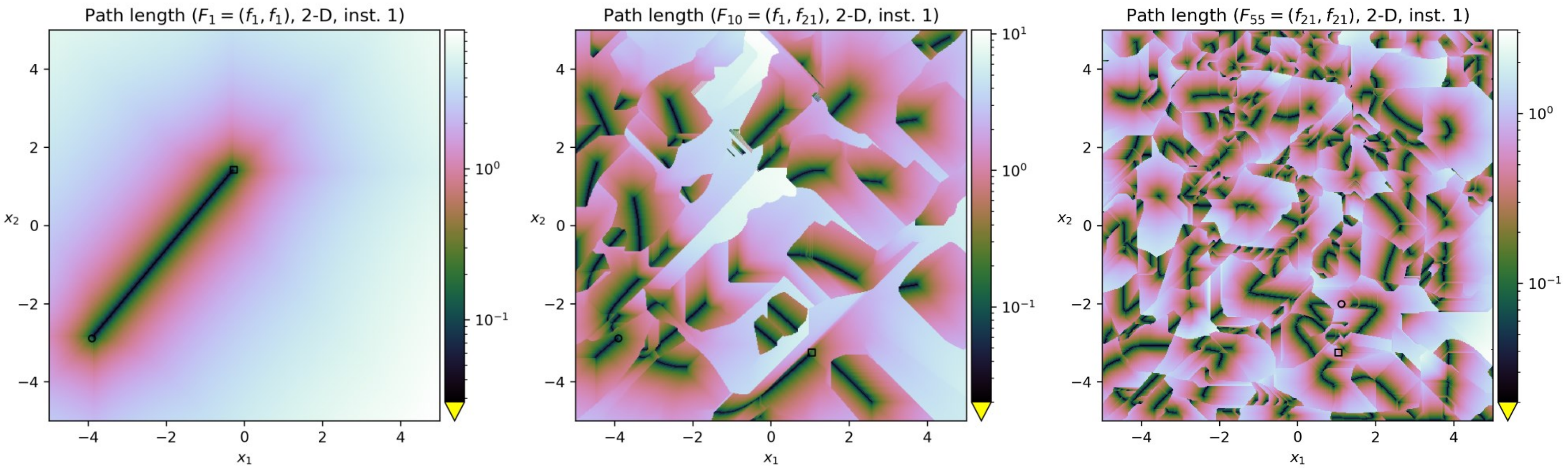
Gradient Path Length

Adapted from [Kerschke and Grimme 2017]

- Compute the ***bi-objective gradient*** for all grid points

$$v = \frac{\nabla f_1(x)}{\|\nabla f_1(x)\|} + \frac{\nabla f_2(x)}{\|\nabla f_2(x)\|}$$

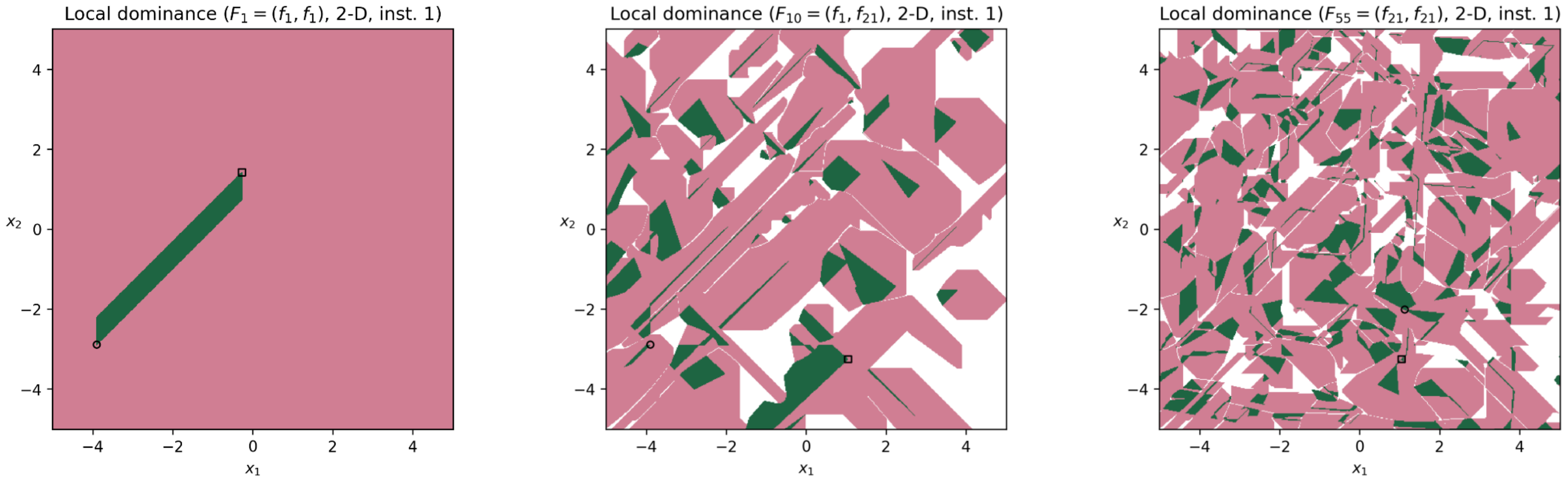
- From a grid point, follow the path in the direction of this gradient
- Visualize the length of the path to the local optimum



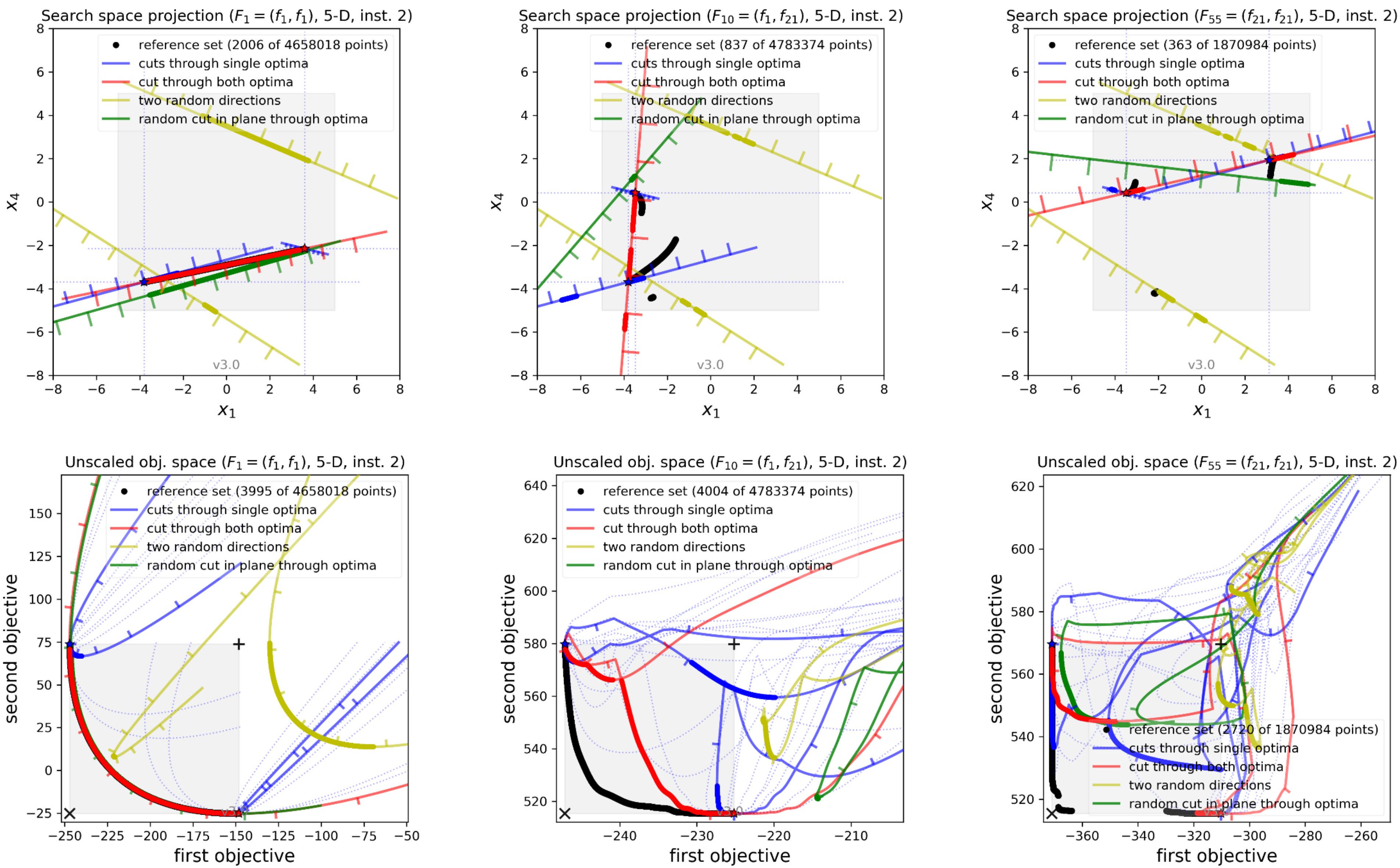
Local Dominance

[Fieldsend et al. 2019]

- Green: Dominance-neutral local optima regions
  - Points that are mutually nondominated with all their 8 neighbors
- Pink: Basins of attraction
  - Points that are dominated by at least one neighbor and whose dominating paths lead to the same green region
- White: Boundary regions
  - Points whose dominating paths lead to different green regions



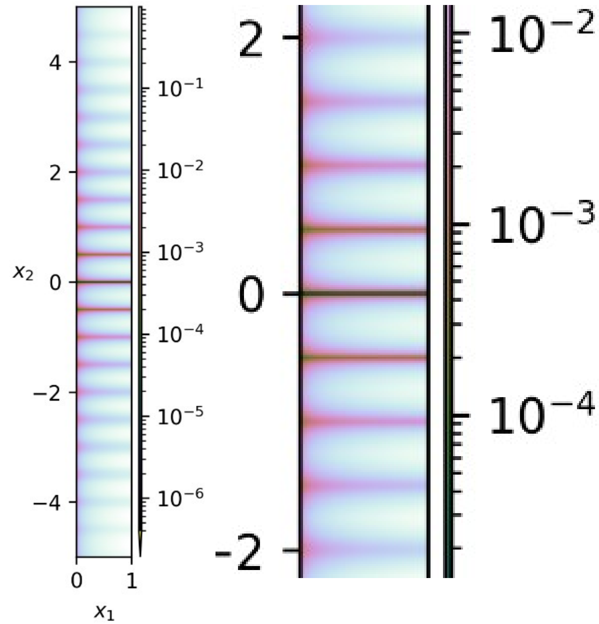
Line Cuts

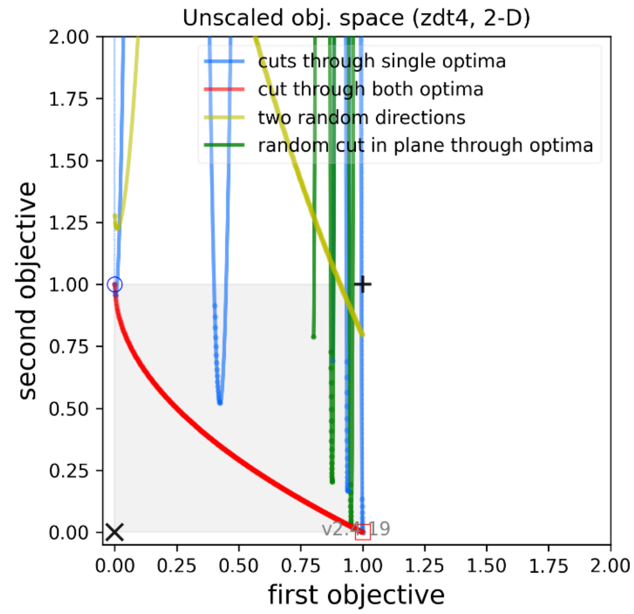


Search space projection ($F_1 = (f_1, f_1)$, 5-D, inst. 2)
reference set (2006 of 4658018 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
v3.0
x_4
x_1
Search space projection ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)
reference set (837 of 4783374 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
v3.0
x_4
x_1
Search space projection ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)
reference set (363 of 1870984 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
v3.0
x_4
x_1
Unscaled obj. space ($F_1 = (f_1, f_1)$, 5-D, inst. 2)
reference set (3995 of 4658018 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
second objective
first objective
Unscaled obj. space ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)
reference set (4004 of 4783374 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
second objective
first objective
Unscaled obj. space ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)
reference set (2720 of 1870984 points)
cuts through single optima
cut through both optima
two random directions
random cut in plane through optima
second objective
first objective

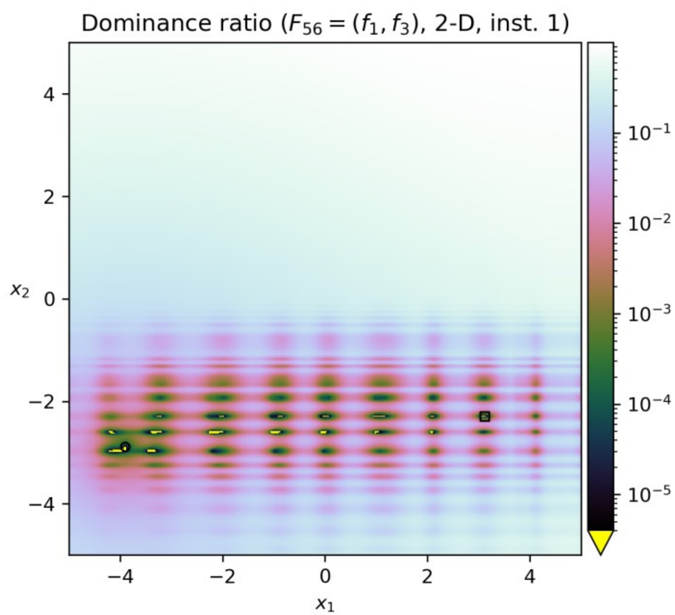
Comparison of Problem Landscapes

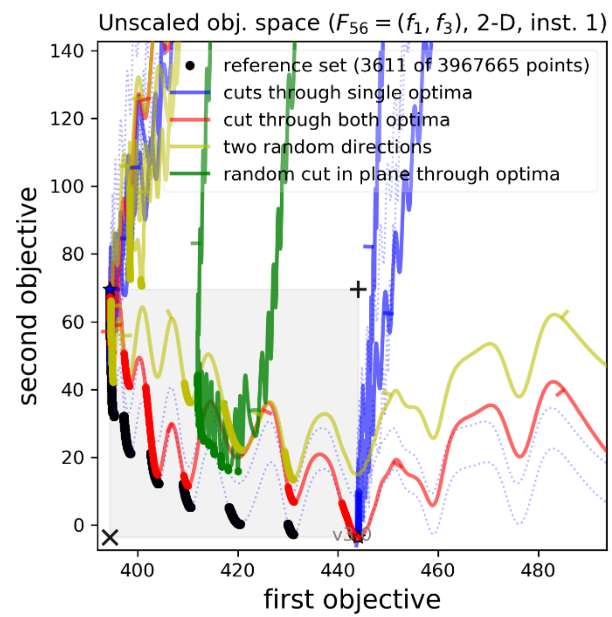
ZDT4





Two problems where both objectives are separable, first is unimodal and second is multimodal





bbob-biobj-ext F_{56}

f_1 Sphere function

f_3 Rastrigin function

Comparison of Problem Landscapes

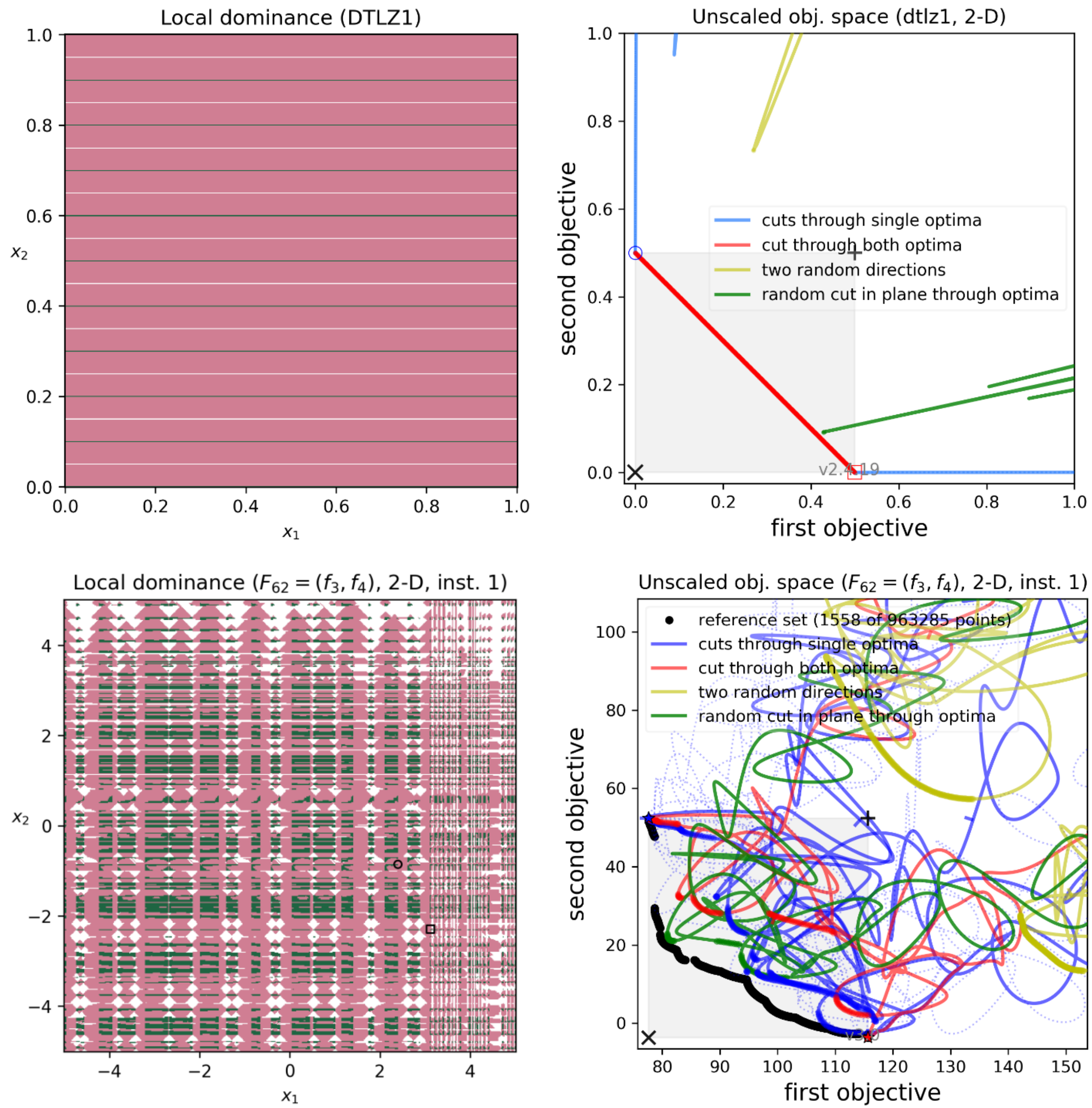
DTLZ1

Two problems where both objectives are separable and multimodal

bbob-biobj-ext F_{62}

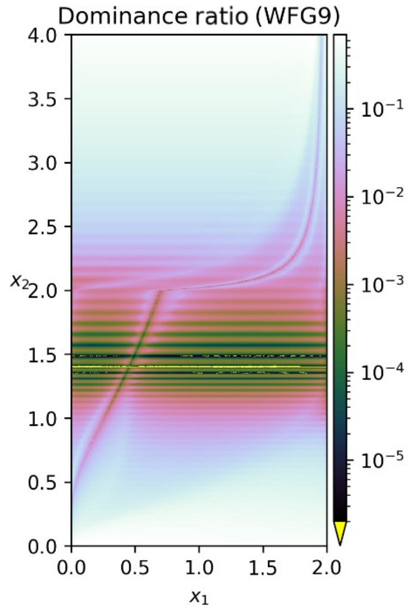
f_3 Rastrigin function

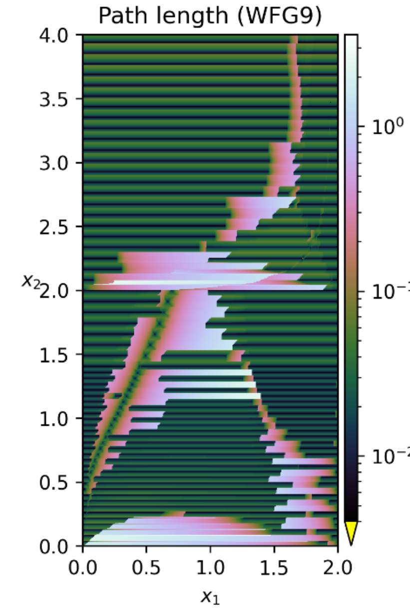
f_4 Skew Rastrigin-Bueche



Comparison of Problem Landscapes

WFG9



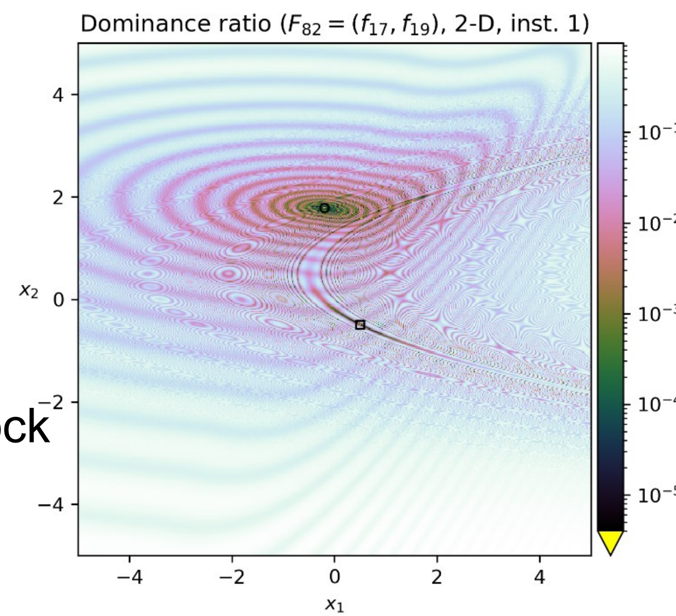


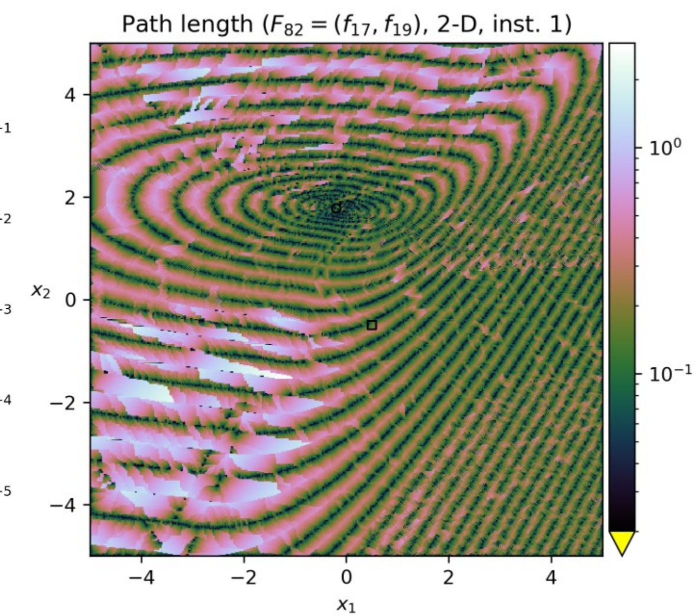
Two problems where both objectives are nonseparable and multimodal

bbob-biobj-ext F_{82}

f_{17} Schaffer F7

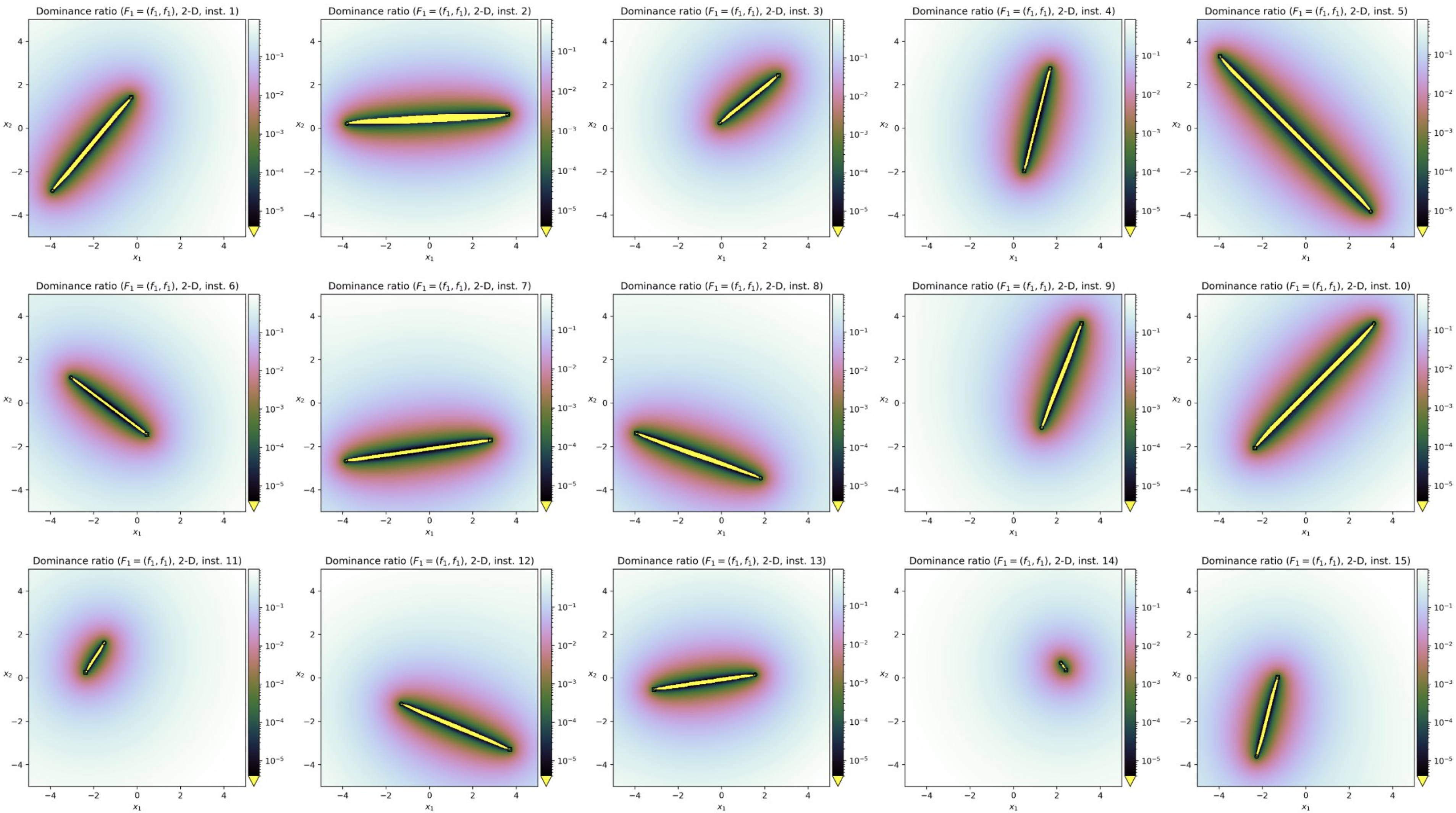
f_{19} Griewank-Rosenbrock



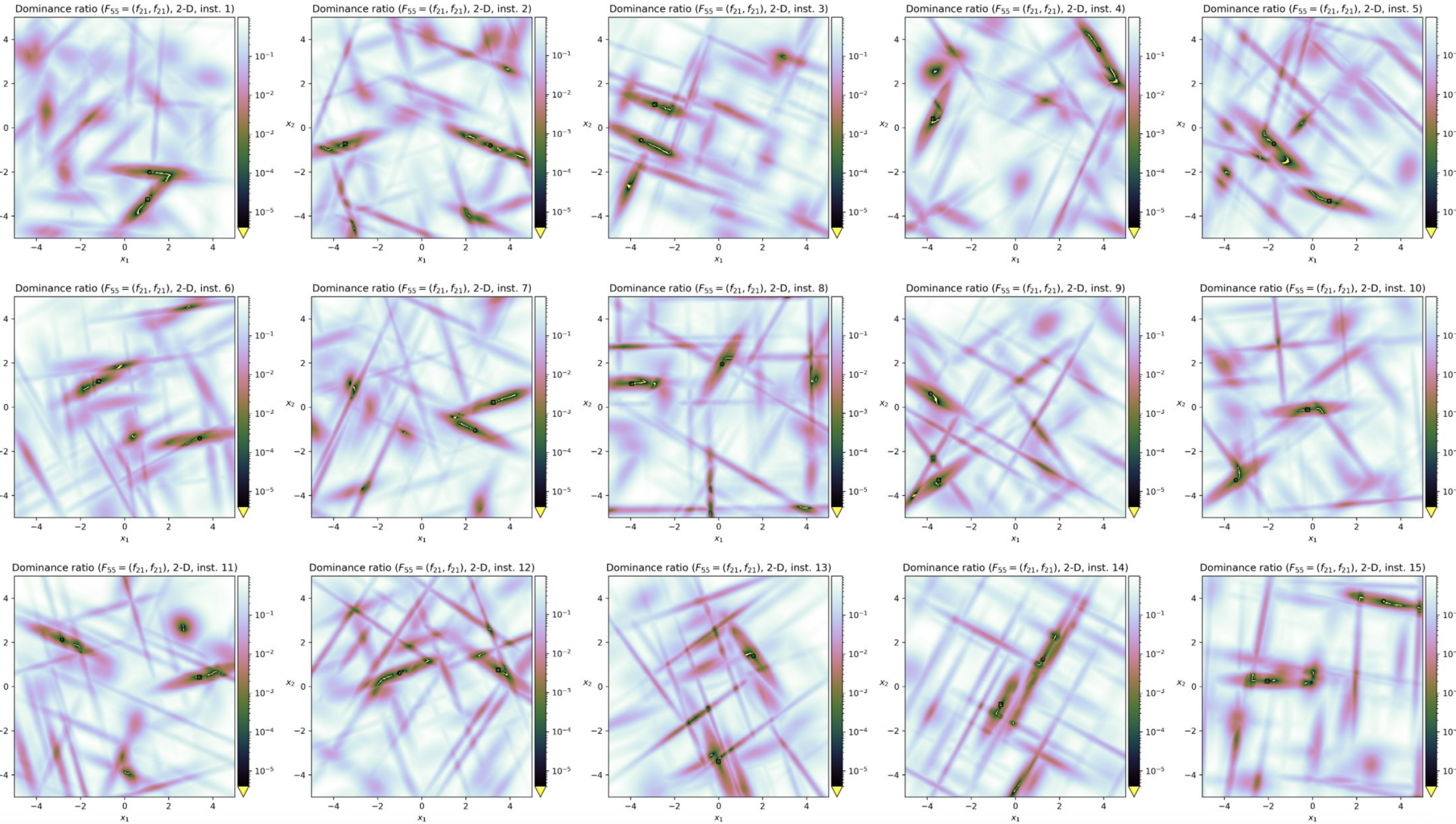


bbob-biobj(-ext) Suite Instances

15 instances of the double sphere problem bbob-biobj F_1



15 instances of the double Gallagher problem bbob-biobj F_{55}



Other artificial problems

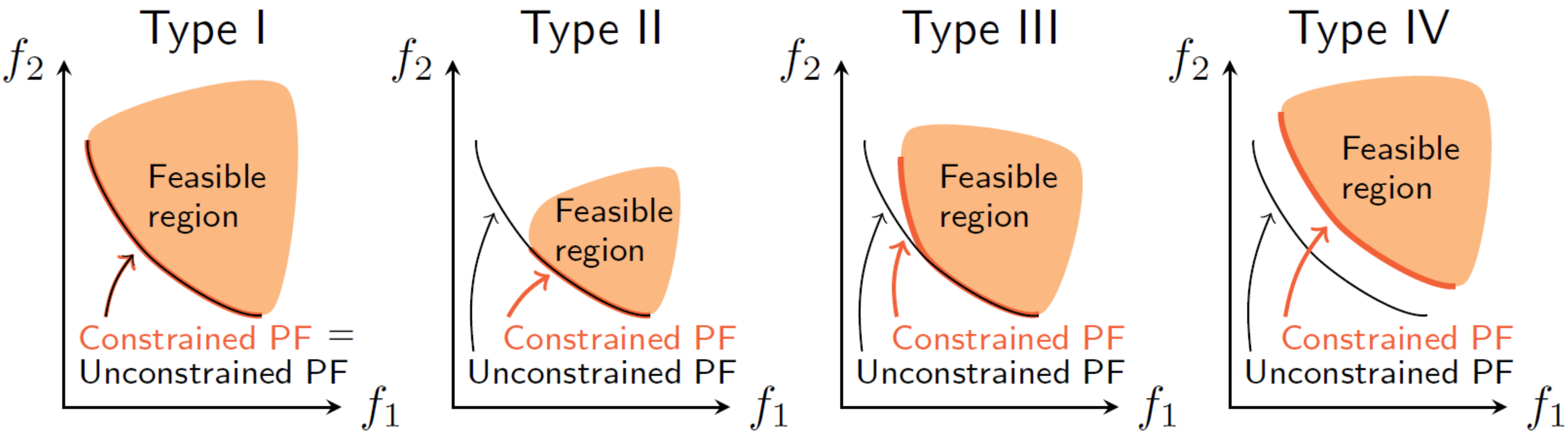
Other Artificial Problems

Suites of multiobjective problems with constraints

- CTP [Deb et al. 2001]
- CF [Zhang et al. 2009]
- C-DTLZ [Jain and Deb 2014]
- NCTP [Li et al. 2016]
- DC-DTLZ [Li et al. 2019]
- LIR-CMOP [Fan et al. 2019]
- MW [Ma and Wang 2019]
- DOC [Liu and Wang 2019]
- DAS-CMOP and DAS-CMaOP [Fan et al. 2020]
- Eq-DLTZ and Eq-IDTLZ [Cuate et al. 2020]
- CLSMOP [He et al. 2021]

Problem types

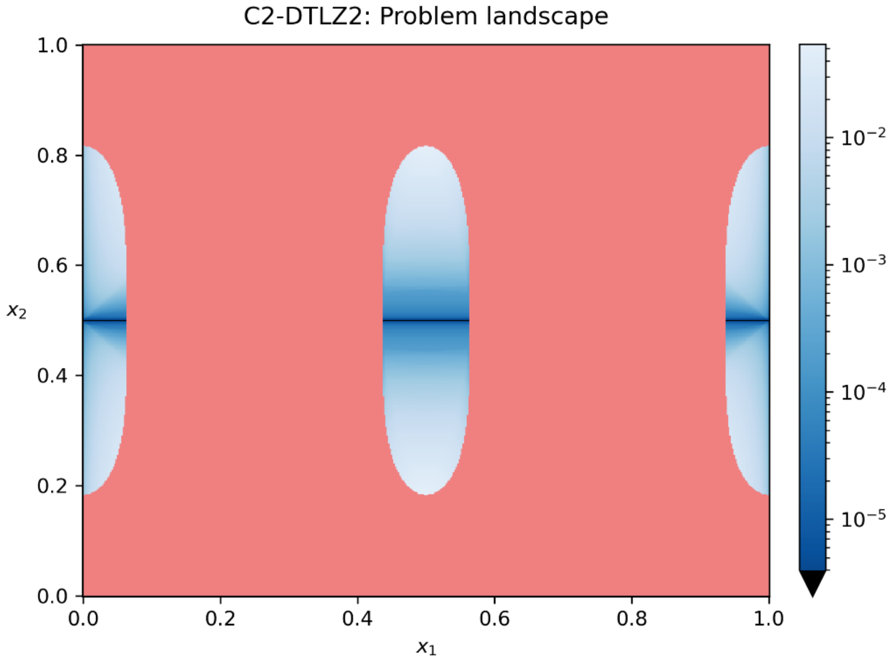
Depending on how the constraints affect the Pareto set/front

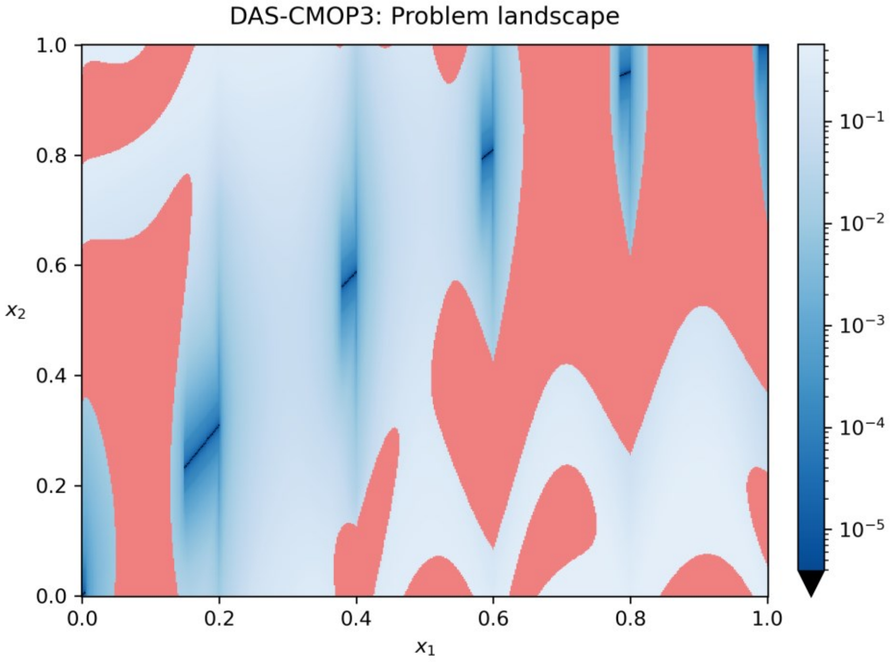


Adapted from [Ma and Wang 2019]

Problems of Type I not useful for benchmarking constraint handling techniques

Analysis of multiobjective problems with constraints





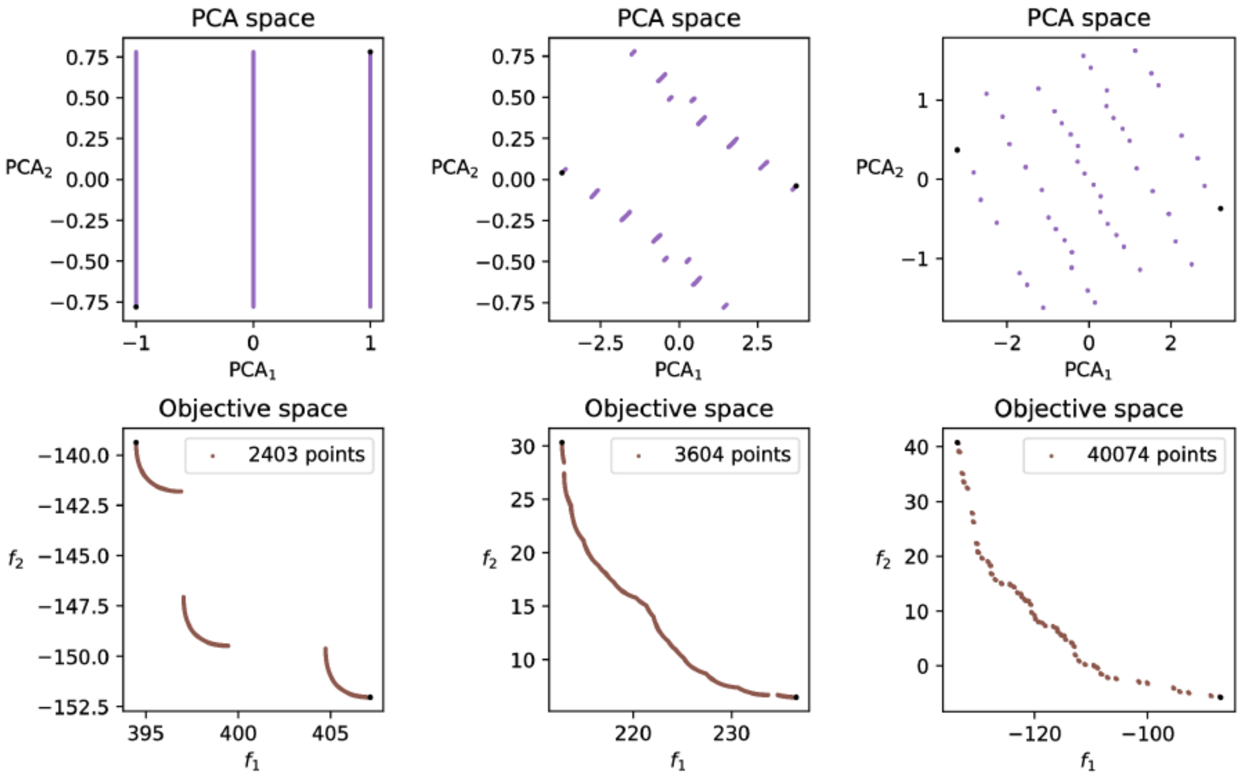
Analysis and visualization of multiobjective problems with constraints:
https://vodopijaaljosa.github.io/cmop-web/

Tutorial on Multiobjective optimization in the presence of constraints:
https://dis.ijs.si/filipic/wcci2022tutorial/

Suites of multiobjective mixed-integer problems

- Exeter suite of 6 problems constructed with the bottom-up approach [McClymont and Keedwell 2011]
- bbob-biobj-mixint suite of 92 bi-objective problems [Tušar et al. 2019]

Pareto set and front approximations for three different instances of the double sphere function



Real-world problems

v0.1

Real-World Problems

Individual problems (white box)

- Water resource planning problem with 3 variables, 5 objectives and 7 constraints [Musselman and Talavage 1980]
- Two bar truss design problem with 2 variables, 2 objectives and 2 constraints [Rao 1987]
- Vibrating platform design problem with 3 variables, 2 objectives and 5 constraints [Ray et al. 2001]
- Welded beam design problem with 4 variables, 2 objectives and 5 constraints [Ray and Liew 2002]
- Multi-speed gearbox design problem with 10 variables, 2 objectives and 38 constraints [Deb and Jain 2003]
- Car side impact problem with 2 variables, 2 objectives and 2 constraints [Jain and Deb 2014]
- …

Real-World Problems

Individual problems (black box)

- Radar waveform design problem with a varying number of variables and 9 objectives [Hughes 2007]
- HBV problem of calibrating the HBV rainfall-runoff model with 14 variables and 4 objectives [Reed et al. 2013]
- MAZDA car structure design problem with 222 integer variables, 2 objectives and 54 constraints [Kohira et al. 2018]
- Lunar lander landing site selection problem with 2 variables, 3 objectives and 2 constraints [JSEC and JAXA 2018]
- Wind turbine design problem with 32 variables, 5 objectives and 22 constraints [JSEC 2019]
- Trappist tour planning problem with 34 mixed-integer variables, 2 objectives and 1 constraint [ESA 2022]

v0.2

Suites of Real-World Problems

Suites of unscalable problems

- DDMOP suite of 7 test problems with a different number of variables (5–17) and objectives (2–10) [He et al. 2020]
- Two suites of previously published problems [Tanabe and Ishibuchi 2020]
  - RE suite of 16 test problems with a different number of variables (2–7) and objectives (2–9)
    - 11 continuous, 1 integer, 4 mixed-integer
  - CRE suite of 8 test problems with constraints a different number of variables (3–7) and objectives (2–5)
    - 6 continuous, 1 integer, 1 mixed-integer
- RCM suite of 50 problems with a different number of variables (2–34), objectives (2–5) and constraints (1–29) [Kumar et al. 2021]

v0.5

Suites of Real-World Problems

Suites of scalable problems

- Heat exchanger design problem with scalable variables and 1 or 2 objectives [Daniels et al. 2018]
- Suite of 3 bi-objective TopTrumps problems in multiple dimensions and instances [Volz et al. 2019]
- Suite of 26 bi-objective MarioGAN problems in multiple dimensions and instances [Volz et al. 2019]
- MODAct suite of 20 problems with 2+6k variables, various number of objectives (2–5) and constraints (7–10) [Picard and Schiffmann 2021]
- Framework with scalable pathfinding problems (5 different objectives) [Weise and Mostaghim 2022]

Conclusions

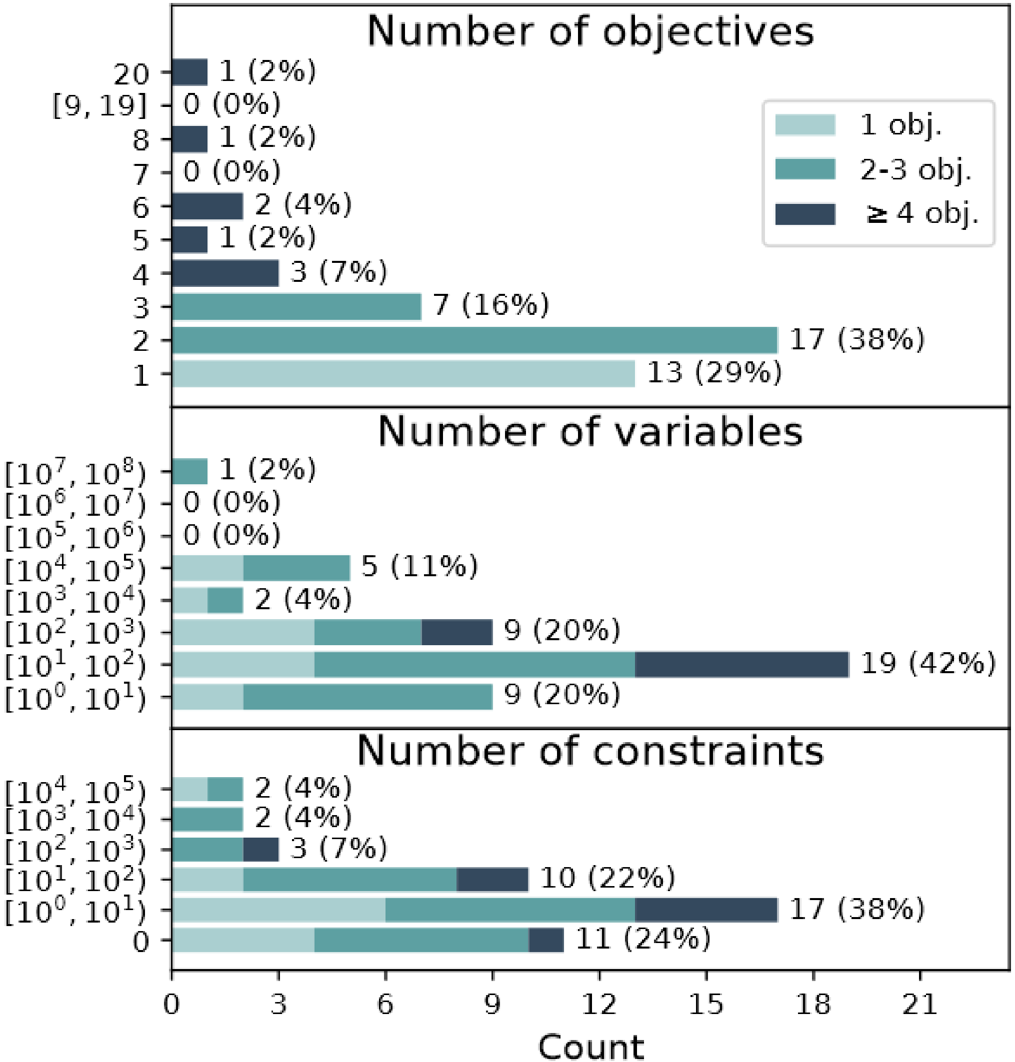
We should think about the usefulness of our research

Results of a questionnaire on the properties of real-world problems has shown their diversity [van der Blom et al. 2023]

https://sites.google.com/view/macoda-rwp/home

Most research is done on continuous unconstrained problems

Although the test problems are scalable, most studies use a fixed number of variables



Conclusions

Problem suites constructed with the bottom-up approach have unrealistic properties

Algorithms are overfitting to these problems (especially the overused DTLZ and WFG) [Ishibuchi et al. 2017]

Using separate functions for the objectives looks like a step in the right direction

Overview

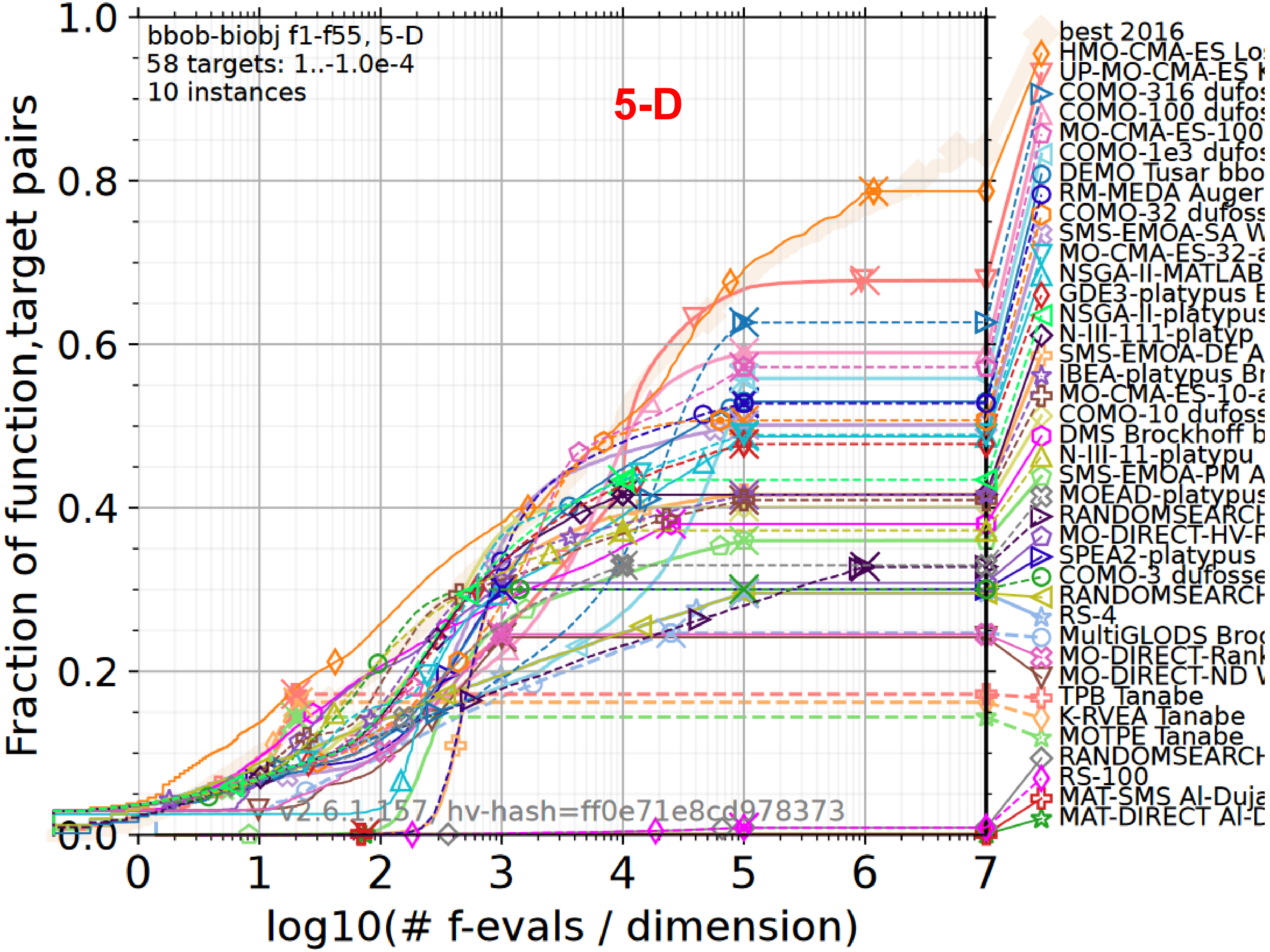
❶ Performance Assessment

❷ Test Problems and Their Visualizations

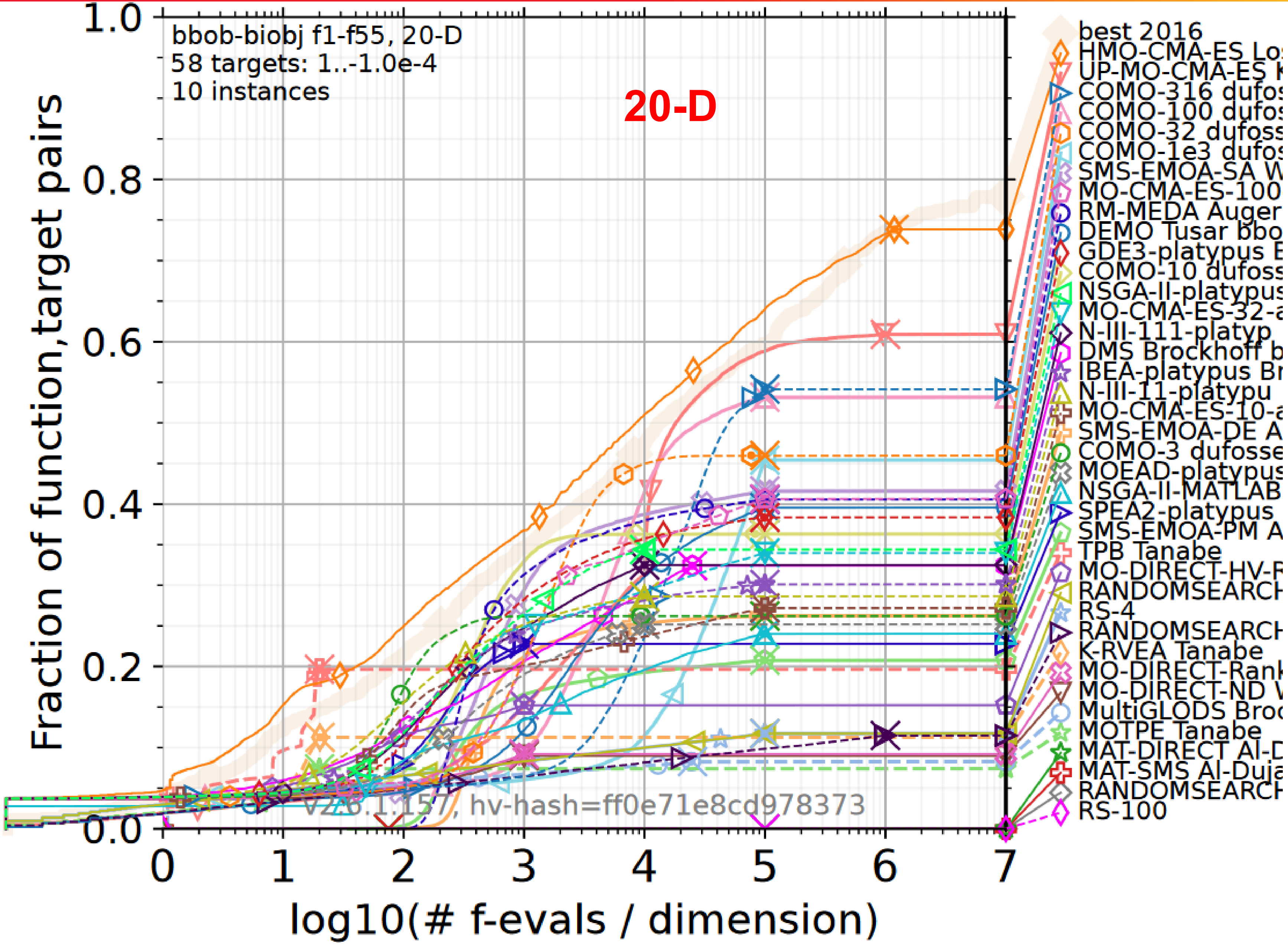
❸ Recommendations from Numerical Results

```
python -m cocopp bbob-biobj*
```

Aggregated Results Over All 55 Functions



Aggregated Results Over All 55 Functions



Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

- many-objective problems
  - problems/suites
  - indicators
  - efficient implementations

Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

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- benchmarking results from more classical approaches

12th GECCO Workshop on Black-Box Optimization Benchmarking (BBOB 2023)

Lisbon, Portugal (hybrid) – July 15-19, 2023

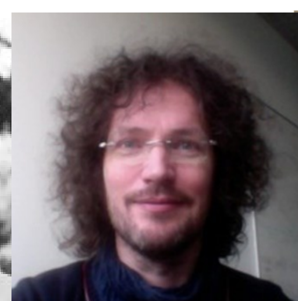
Submit your 8-pages paper on any topic related to benchmarking blackbox optimizers!

Topics of interest (amongst others):

- Noiseless optimization
- Noisy optimization
- Bi-objective optimization
- Mixed-integer optimization
- Large-scale optimization
- Constrained optimization

Submission deadline **April 14, 2023**

Support for automated benchmarking including LaTeX template via the COCO platform.

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anything else than tables for fixed budget

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Thank you!

Supplementary Material

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